What do I need to know as a teacher in order to be able to teach the concept(s)?

+Vocabulary

Addend
- The numbers being added together in an addition question.

Associative Property
- You can add numbers regardless of how the numbers are grouped using parenthesis. Ex. (3 + 5) + 2 is the same as 3 + (5 + 2). It's used when dealing with 3 or more numbers

Commutative Property
- When adding, the order of the addends does not matter. The result is the same. Ex. 3 + 5 is the same as 5 + 3.

Compensation
- A strategy used in mental math in which you change one addend to a multiple of ten and then adjust the other addend to keep the balance. Ex. 37 + 49 is the same as 40 + 46. You added 3 to 37 to make it a multiple of 10, therefore you must remove than extra 3 from the 49.

Constant Difference
- Adding or subtracting the same number to both the subtrahend and the minuend in a subtraction problem does not change the answer. 6 - 3 = 3; 7 - 4 = 3; 8 - 5 = 3

Decomposing Numbers
- Breaking up a number into place value pieces (349 ⇨ 300 + 40 + 9)

Difference
- The result of subtracting two numbers.

Friendly Numbers
- A friendly number is simply a number that is easy to work with. This normally refers to multiples of 10.

Inverse Operation
- "The operation which is 'opposite' mathematically to that being considered. Thus, subtraction is the inverse of addition and vice versa." Source

Minuend

- The first number in a subtraction. The number from which another number (the Subtrahend) is to be subtracted. Ex. 20 - 5 (20 is the minuend)

Partitioning

- Breaking up a number into more usable pieces (349 + 123 ⇨ 349 + 100 + 23)

Subtrahend

- The number subtracted from the minuend Ex. 20 - 5 (5 is the subtrahend)

Sum

- The result of adding two or more numbers together

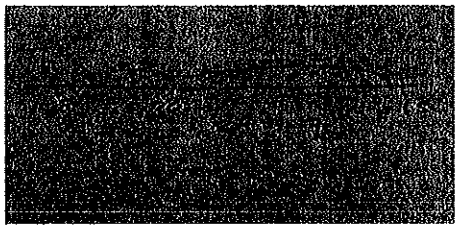
Further Vocabulary Resources:

- Math is Fun - Illustrated Mathematics Dictionary – Mathematics Vocabulary & Illustrations https://www.mathsisfun.com/definitions/index.html
- Mathematics Glossary - *LearnAlberta.ca*
  - Example Link - Associative Property http://www.learnalberta.ca/content/memg/division04/associative%20property/index.html

What do I need to know as a teacher in order to be able to teach the concept(s)?

+Potential Misunderstandings

The Question: (Focusing on the Misconception)	The Follow Up Question that will challenge the assumption.	Background Info
You can't subtract a bigger number from a smaller number. Ex. 3 - 5 is impossible.	If it is 3° C and the temperature drops by 5°, what is the temperature now?	Misunderstanding: Students can struggle with the idea of removing more than you start with. Temperature is an excellent tool for developing this concept appropriately as it is a natural situation where negatives exist. Another opportunity for exploration could focus on wanting to buy an item but not having enough money. Example, you have $10 but the item costs $15. How much money are you short? How much do you have to borrow from your parents in order to buy it? How much do you owe them?"
Subtraction is commutative. It doesn't matter what order you do it in. Ex. 3 - 5 is the same as 5 - 3.	Is "I have $3 and spend $5" the same as "I have $5 and spend $3"?	Misunderstanding: Students learn that addition is commutative so it doesn't matter if you do 2 + 4 or 4 + 2 because you get the same answer. They transfer this concept to subtraction. It is compounded by the misconception that you can not subtract a bigger number from a smaller number.

When "more" shows up in a word problem, I have to add.	Johnny has 10 pencils. Suzy has 15 pencils.How many more pencils does Suzy have?	Misunderstanding: Word problems have cue words that will tell you what operation to use. Ex. "More" means you have to add. Cue words on their own do not tell you what type of operation to use. Students must carefully analyze the problem, using all of the information provided in order to determine the appropriate operation(s) to use.
In order to solve the following question, you must subtract. "Suzy has 15 pairs of shoes. She donates 8 pairs to Value Village. How many pairs does she have left?"	Johnny solves this question as 	Misunderstanding: Students see addition and subtraction as discrete operations rather than inverse operations. However, all subtraction sentences can be solved by thinking additively.

Below you will find outcomes from the elementary program of studies related to additive thinking as the EMPL team sees it. Other outcomes may be closely related but were not included in this project.

+Kindergarten

N 5	Compare quantities 1 to 10, using one-to-one correspondence. [C, CN, V] **This outcome is identified as "Pre-Operations"*

+Grade One

N 2	Subitize (recognize at a glance) and name familiar arrangements of 1 to 10 objects or dots. [C, CN, ME, V] **This outcome is identified as "Pre-Operations"*
N 5	Compare sets containing up to 20 elements, using: • referents • one-to-one correspondence to solve problems. [C, CN, ME, PS, R, V]
N 8	Identify the number, up to 20, that is: • one more • two more • one less • two less than a given number. [C, CN, ME, R, V] **This outcome is identified as "Pre-Operations"*
N 9	Demonstrate an understanding of addition of numbers with answers to 20 and their corresponding subtraction facts, concretely, pictorially and symbolically, by: • using familiar mathematical language to describe additive and subtractive actions • creating and solving problems in context that involve addition and subtraction • modeling addition and subtraction, using a variety of concrete and visual representations, and recording the process symbolically. [C, CN, ME, PS, R, V]

N 10	Describe and use mental mathematics strategies, such as: • counting on and counting back • making 10 • using doubles • Thinking addition for subtractionfor basic addition facts and related subtraction facts to 18. [C, CN, ME, PS, R, V] *Clarification:* Understand and apply strategies for addition and related subtraction facts to 18. Recall addition and related subtraction facts to 5.

+Grade Two

N 8	Demonstrate and explain the effect of adding zero to, or subtracting zero from, any number. [C, R]
N 9	Demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems that involve addition and subtraction • using the commutative property of addition (the order in which numbers are added does not affect the sum) • using the associative property of addition (grouping a set of numbers in different ways does not affect the sum) • explaining that the order in which numbers are subtracted may affect the difference. [C, CN, ME, PS, R, V]
N 10	Apply mental mathematics strategies, such as: • using doubles • making 10 • ~~one more, one less~~ • ~~two more, two less~~ • building on a known double • thinking addition for subtraction for basic addition facts and related subtraction facts to 18. [C, CN, ME, PS, R, V] *Clarification:* Understand and apply strategies for addition and related subtraction facts to 18. Recall addition and related subtraction facts to 10.

N 6	Describe and apply mental mathematics strategies for adding two 2-digit numerals, such as: • adding from left to right • taking one addend to the nearest multiple of ten and then compensating • using doubles. [C, CN, ME, PS, R, V]
N 7	Describe and apply mental mathematics strategies for subtracting two 2-digit numerals, such as: • taking the subtrahend to the nearest multiple of ten and then compensating • thinking of addition • using doubles. [C, CN, ME, PS, R, V]
N 8	Apply estimation strategies to predict sums and differences of two 2-digit numerals in a problem-solving context [C, ME, PS, R]
N 9	Demonstrate an understanding of addition and subtraction of numbers with answers to 1000 (limited to 1-, 2- and 3-digit numerals), concretely, pictorially and symbolically, by: • using personal strategies for adding and subtracting with and without the support of manipulatives • creating and solving problems in context that involve addition and subtraction of numbers. [C, CN, ME, PS, R, V]
N 10	Apply mental mathematics strategies and number properties, such as: • using doubles • making 10 • using the commutative property • using the property of zero • thinking addition for subtraction or basic addition facts and related subtraction facts to 18. [C, CN, ME, PS, R, V]
P/R 4	Solve one-step addition and subtraction equations involving a symbol to represent an unknown number. [C, CN, PS, R, V]

+Grade Four

N 3

Demonstrate an understanding of addition of numbers with answers to 10 000 and their corresponding subtractions (limited to 3- and 4-digit numerals) by:

- using personal strategies for adding and subtracting
- estimating sums and differences
- solving problems involving addition and subtraction.

[C, CN, ME, PS, R]

N 5

Describe and apply mental mathematics strategies, such as:

- skip counting from a known fact
- ~~using doubling or halving~~
- ~~using doubling or halving and adding or subtracting one more group~~
- ~~using patterns in the 9s facts~~
- ~~using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts.~~

[C, CN, ME, R]

N 11

Demonstrate an understanding of addition and subtraction of decimals (limited to hundredths) by:

- using personal strategies to determine sums and differences
- estimating sums and differences using mental mathematics strategies to solve problems.

[C, ME, PS, R, V]

P/R 6

Solve one-step equations involving a symbol to represent an unknown number.
[C, CN, PS, R, V]

+Grade Five

N 11

Demonstrate an understanding of addition and subtraction of decimals (limited to thousandths).
[C, CN, PS, R, V]

+Grade Six

N 2

Solve problems involving whole numbers and decimal numbers.
[ME, PS, T]

Evidence of Understanding

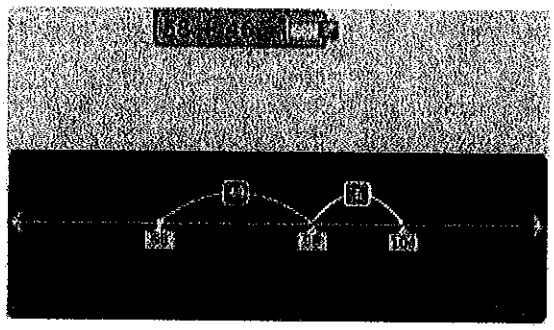
+Big Idea #1

Once students trust "the count", they can flexibly manipulate numbers in order to make solving problems easier by:

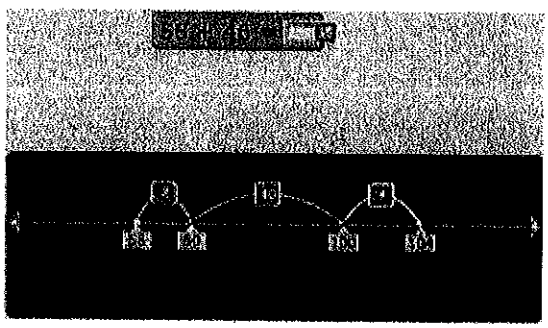
- Using Parts and Wholes
- Decomposing / recomposing
- Partitioning
- Compensating
- Using constant difference

What to look for, what might be evidence of understanding?

- Students manipulate numbers using a variety of strategies to solve questions.
  - 7 + 8 = 7 + 7 + 1
  - 12 + 9 = 10 + 11
  - 99 + 24 = 100 + 23
  - 324 + 138 = 300 + 100 + 20 + 30 + 4 + 8



- Dreambox tool (dreambox.com)

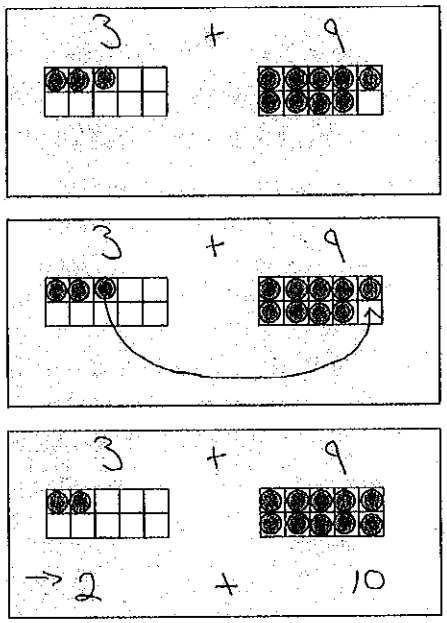


- Dreambox tool (dreambox.com)
- Once students work with simplistic numbers, they can transfer that knowledge to larger numbers

Support

If a student solves a subtraction question only using subtraction strategies, ask them if they could solve it by adding.

If a student does not realize they can manipulate numbers in order to make adding and subtracting easier, you could provide them with ten frames activities that allow them to visualize this process.



+Big Idea #2

Students use mathematical reasoning to build connections between inverse problems.

What to look for, what might be evidence of understanding?

- Students manipulate numbers in order to solve similar questions using both additive and subtractive actions.
    - Example: Simpler Question: 10 - 4
        - Subtraction: Students may solve it using a variety of strategies, such as with Cuisenaire Rods

Question 

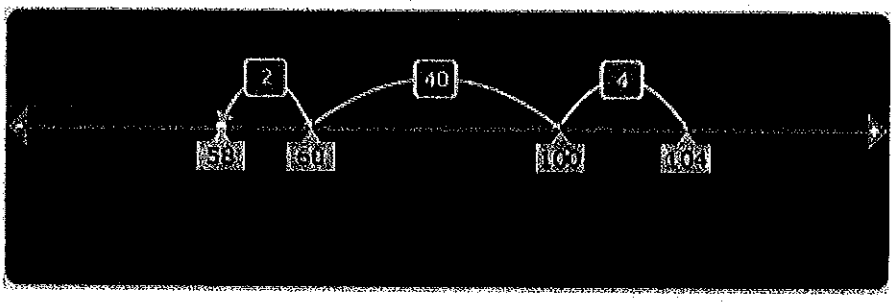
Answer 

NRICH: http://nrich.maths.org/4348

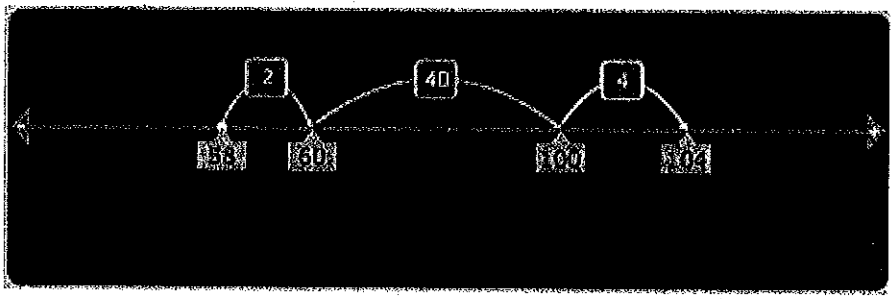
- Addition: Students may solve it using a variety of strategies, such as with Cuisenaire Rods

NRICH: http://nrich.maths.org/4348

- Example: More Challenging Question: 104-46
  - Subtraction: Students may solve it using a variety of strategies, such as with a number line.



  - Addition: Students may solve it using a variety of strategies, such as with a number line.



- Once students work with simplistic numbers, they can transfer that knowledge to larger numbers

Support:
If a student solves a subtraction question only using subtraction strategies, ask them if they could solve it by adding.

+Big Idea #3

Addition is not just adding. It's subtraction as well as it deals with questions where the start, change or result is unknown. It is joining, separating and comparing.

What to look for, what might be evidence of understanding?

- Students solve a given problem involving a missing addend, minuend or subtrahend, describing the strategy used.
    - e.g. Paul has some baseball cards. His friend gave him 4 more. Now Paul has
    - e.g. Sylvia had 5 cookies. She gave some to her friend. Now she has 2 left. 42 cards. How many did Paul have to begin with? How many cookies did she give to her friend?

Support:

Provide students with many opportunities to experience several types of questions. The "categories" provided below contain models showing the variety of question types that students should be exposed to. Students are not expected to identify the category of a question.

- **Combine (P - P - W)**
  Combine (Part - Part - Whole) problems involve distinctly different sets being combined. There is no direct or implied action. You are asked to consider the two SUBSETS as one large set but neither set changes in the process.
    - Whole unknown: Sally has $15 in bills and $5 in coins. How much does she have altogether?
    - Part unknown: Sally has $32. $15 are in bills and the rest is in coins. How much is in coin?

- **Change (Join)**
  Change (Join) problems involve two subsets made up of the same items being joined to make a combined set.
    - Result unknown: Sally has $35.25. She earns $58.85. How much does she have at the end of the day?
    - Start unknown: Sally has a few dollars. John has $7. Together they have $13. How much does Sally have?
    - Change unknown: Sally has $28. How much more money does she need to save if she wants to buy a $37 game?

- **Change (Seperate)**
  Change (Separate) problems involve a large homogeneous set being separated into two subsets made up of the same items.
    - Result unknown: Sally has $57. She gives $32 to pay her mother back. How much money does she have left?
    - Start unknown: Sally has some money in her wallet. She spends $15 at the store. She has $41 left. How much money did she start with?
    - Change unknown: Sally has $28. She buys a gift. She is left with $20. How much was the gift?

- **Compare**
  Compare problems involves a comparison between two quantities.
  - Compare Quantity unknown (Type 1): Sally has $75. She has $30 more than John. How much money does John have?
  - Compare Quantity unknown (Type 2): Sally has $42. John has $15 more than Sally. How much money does John have?
  - Difference unknown: Sally has $5.25. John has $3.90. How much more does Sally have than John?

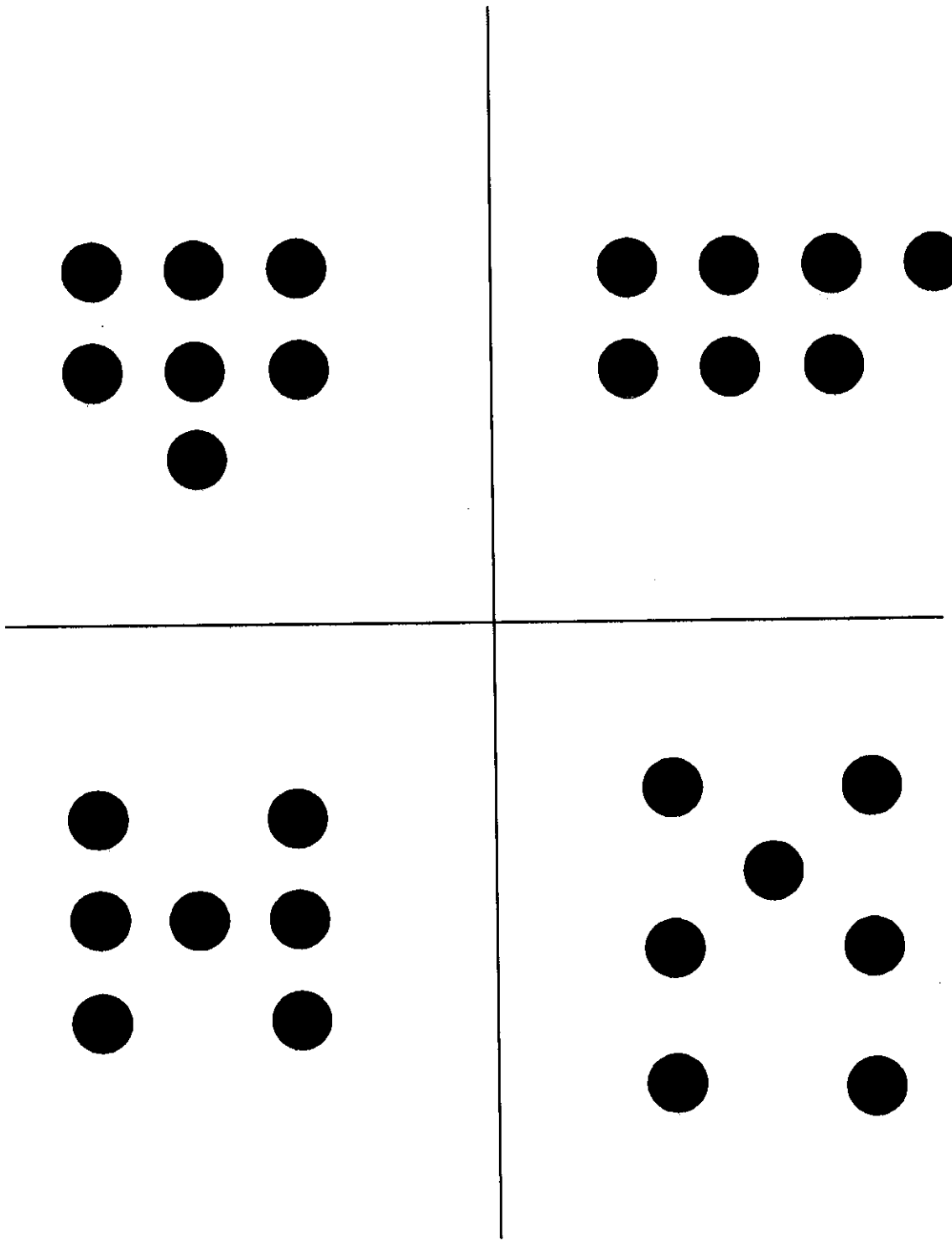
Support
Provide students with a variety of tools such as open number lines, Cuisenaire rods, etc. to help them visualize when solving these questions.

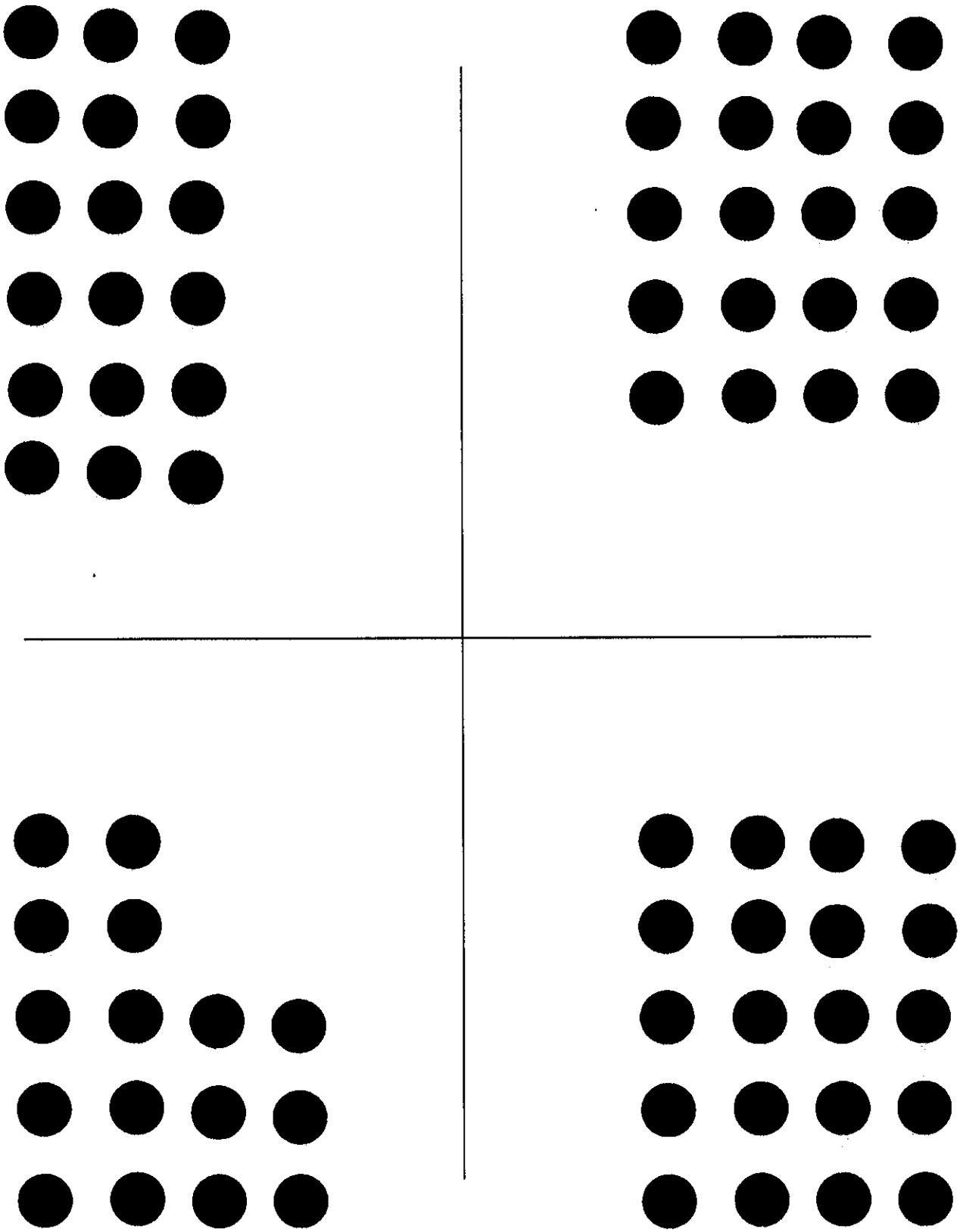
+Big Idea #4

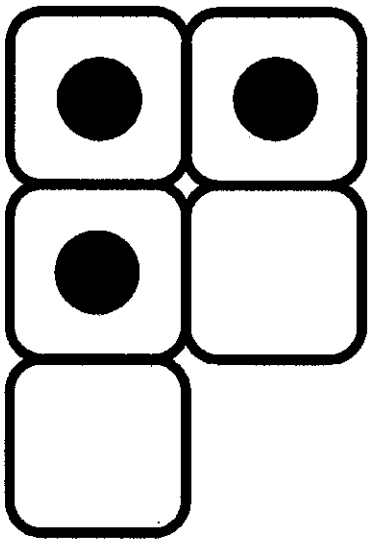
Subtraction is not just "take away". It is also a comparison (how many more, how many less, what is the difference?).

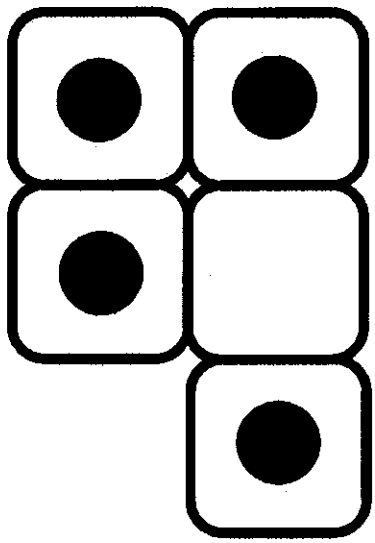
What to look for, what might be evidence of understanding?
- Students think of the subtraction symbol as "how far apart are they?" or "the space between numbers" in addition to "take away".
- Students can look at a direct comparison to find the difference.
  - E.g. Lisa and Mary are growing bean plants in class. Lisa's plant is 7 cm tall. Mary's plant is 12 cm tall. Who's plant is taller and by how much? How do you know?

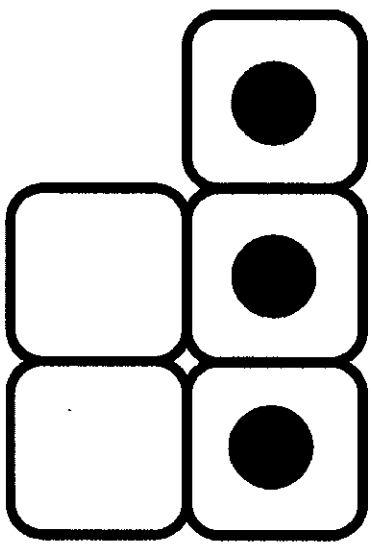
Teacher Support
This website further discusses the complexity of finding sums and differences: Top Drawer Teachers
This document provides teaching guidance for subtraction. LearnTogether.org.uk

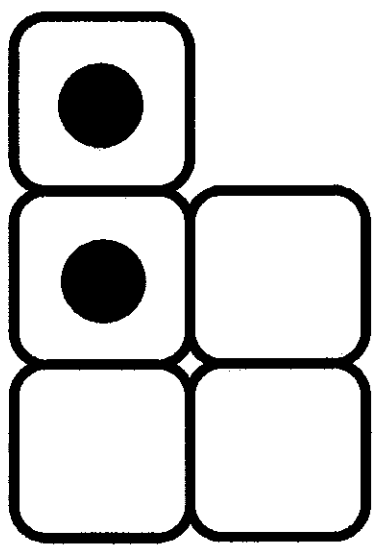


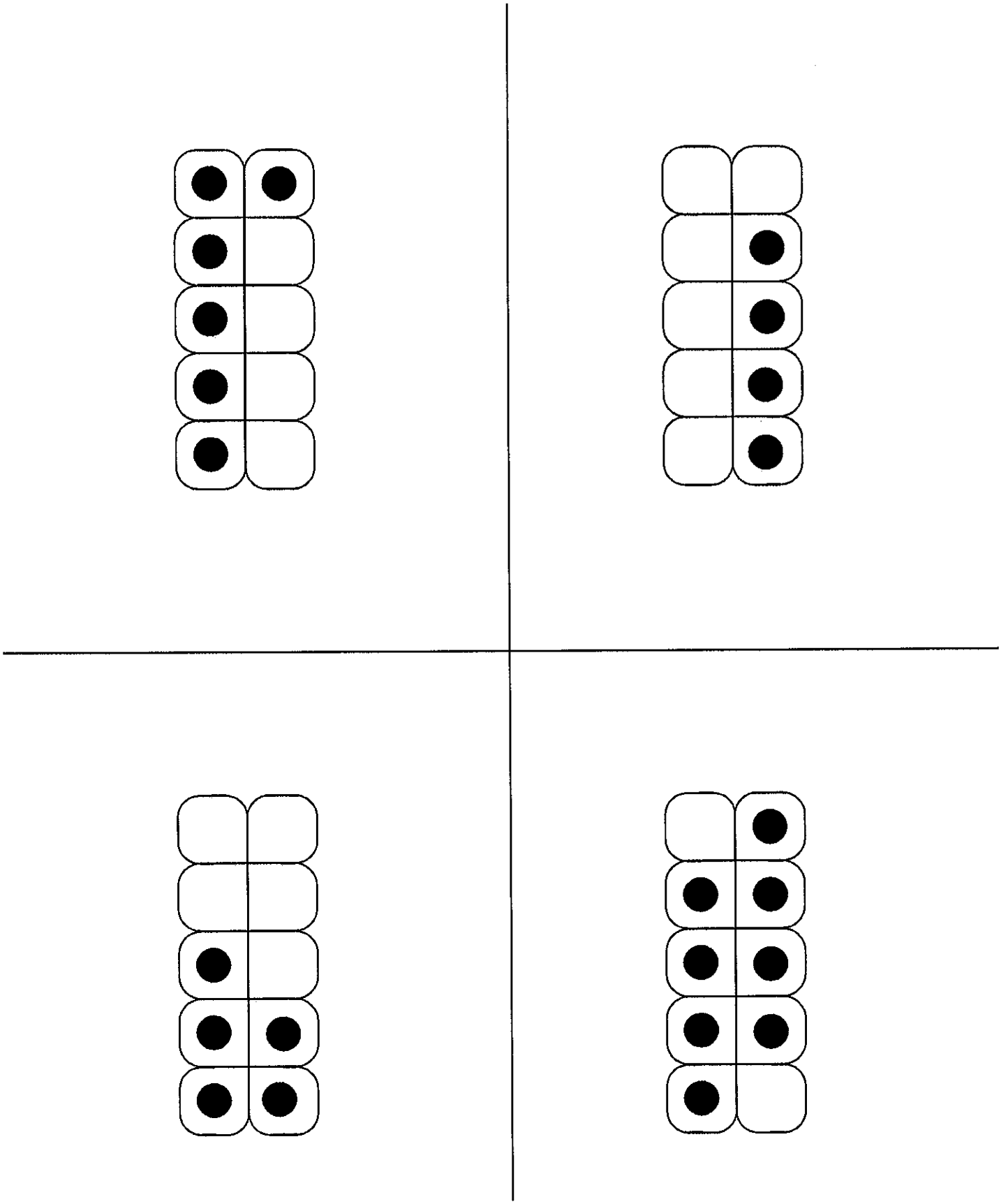


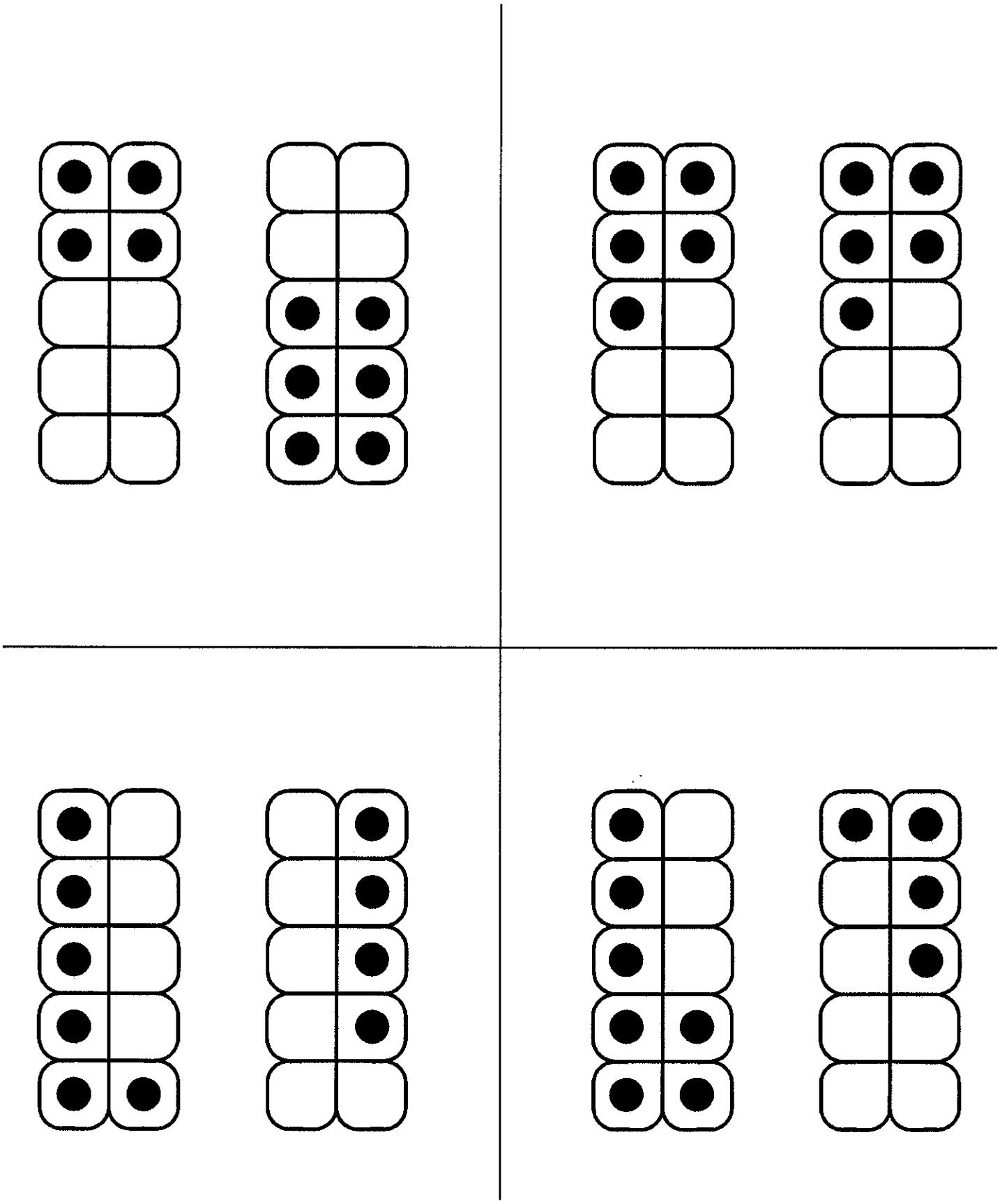


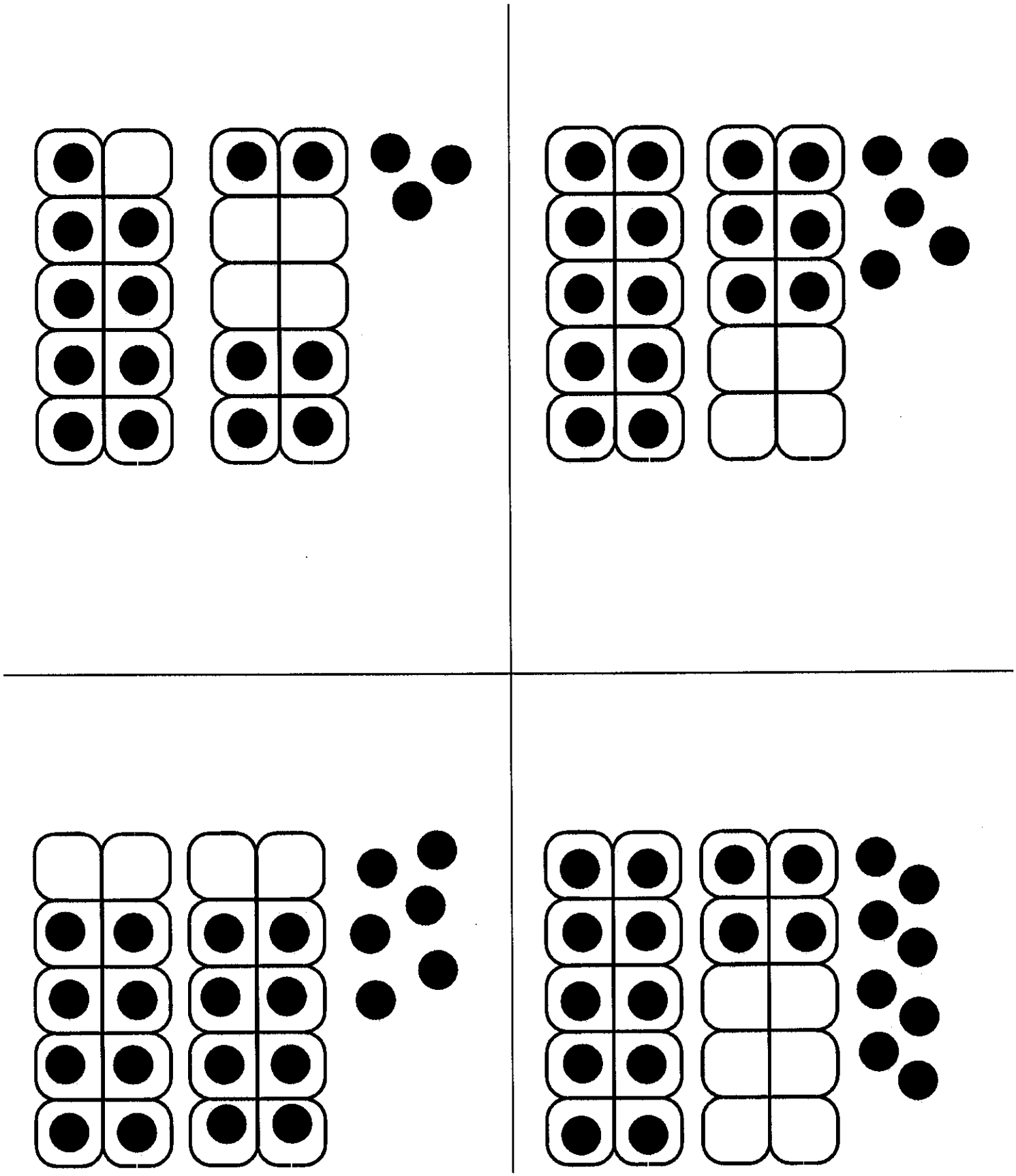












3
7

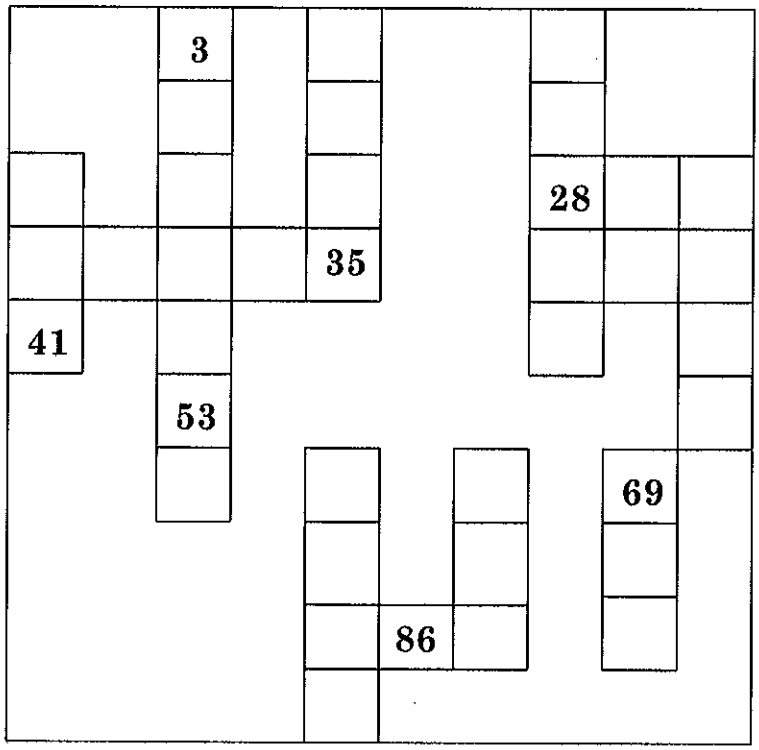
8
3

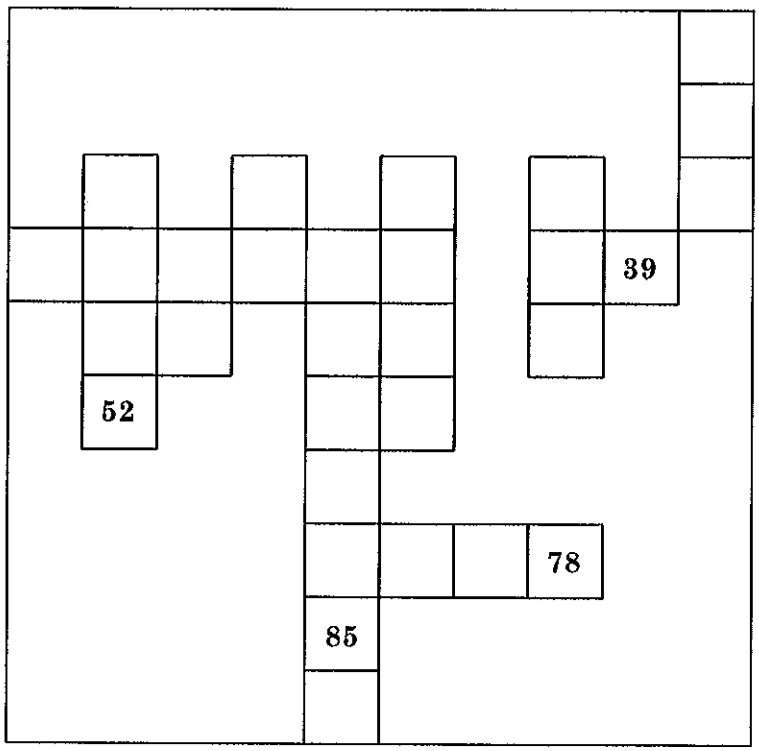
3
3

2
7

45
26
32
50
29
15
25
16

Hundreds Boards





Math Squares

3	2
5	7

_____

8	9
	2

20

7	8
3	

20

9	2
8	

30

5	7
5	5

_____

4	8
	6

28

Math Squares

17	22
28	10

_____

33	27
	28

90

19	24
6	24

_____

30	31
19	

90

	19
11	25

55

9	14
16	

55

Math Squares

27.50	6.00
6.00	10.50

_____

15.50	24.75
5.25	

50.00

3.75	2.25
	1.50

10.00

1.25	
7.50	3.75

60.00

140.00	37.50
2.50	60.00

_____

	60.00
25.50	24.50

130.00

Math Squares

$\frac{1}{2}$	5
4	$\frac{1}{2}$

—

$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	

4

$5\frac{1}{2}$	5
9	$4\frac{1}{2}$

—

$2\frac{1}{4}$	7
8	$6\frac{3}{4}$

—

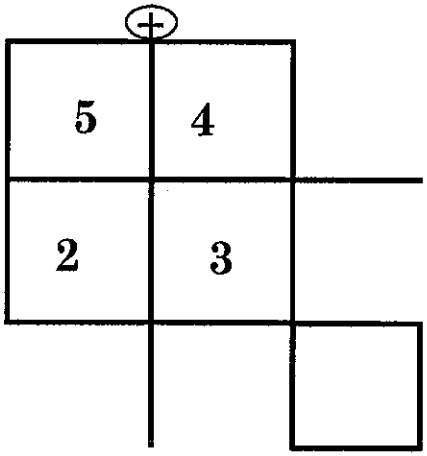
$\frac{1}{4}$	2
	$\frac{3}{4}$

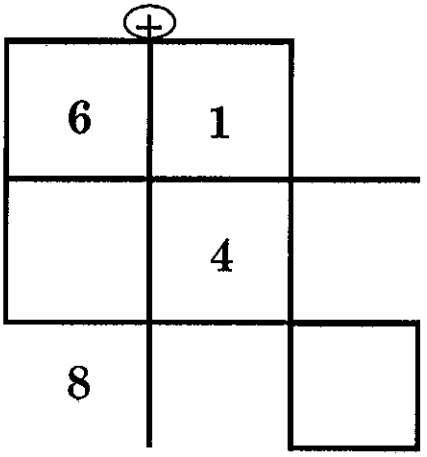
$5\frac{1}{4}$

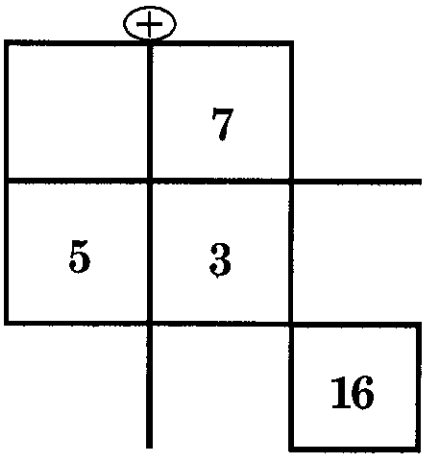
$\frac{3}{4}$	$\frac{1}{2}$
	$\frac{1}{4}$

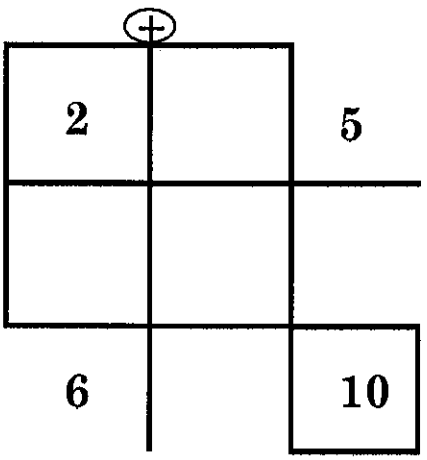
$4\frac{1}{2}$

Two Ways

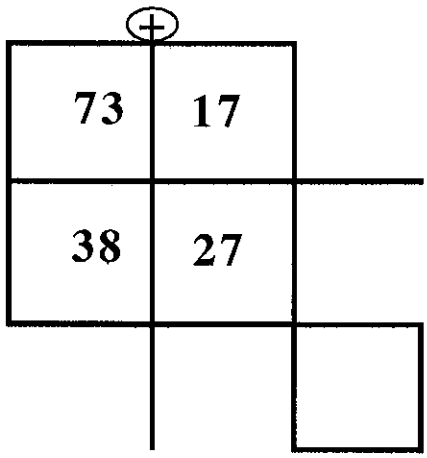


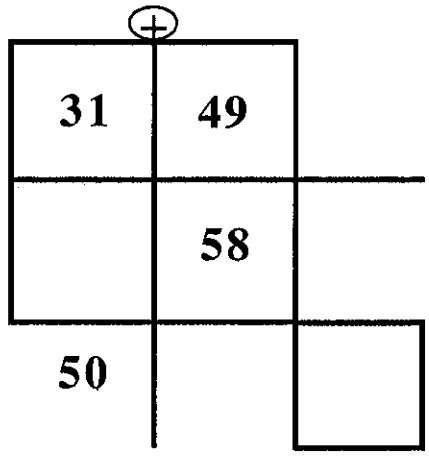


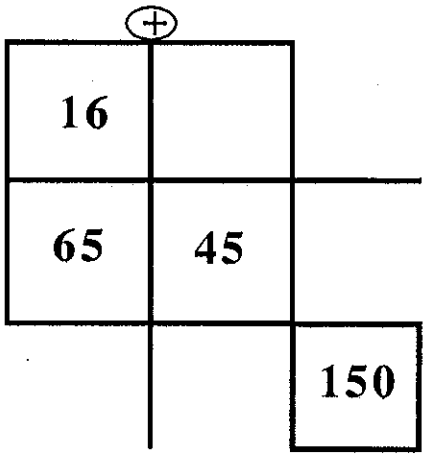


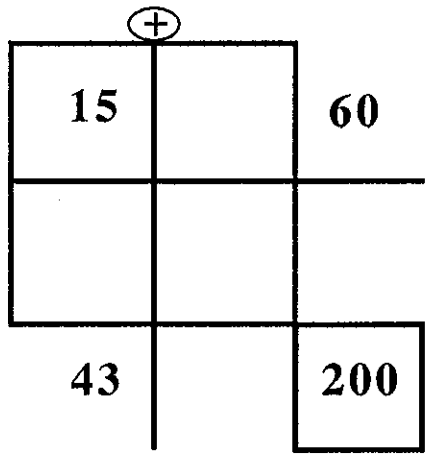


Two Ways



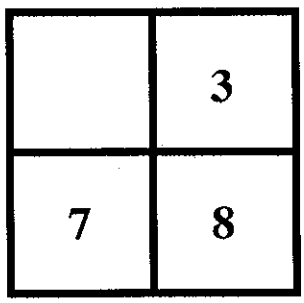






Integer Math Squares # 1

1.



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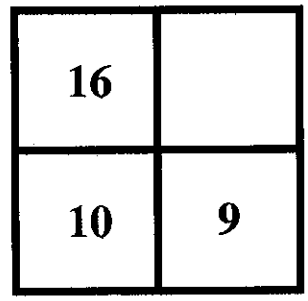

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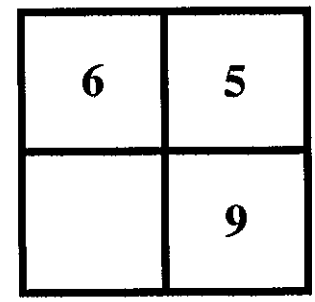
3.



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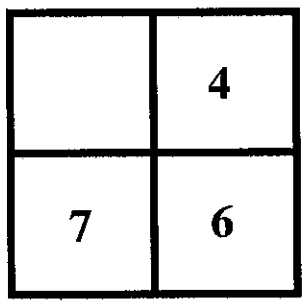

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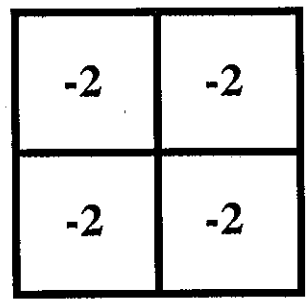

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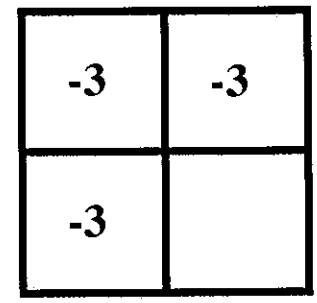

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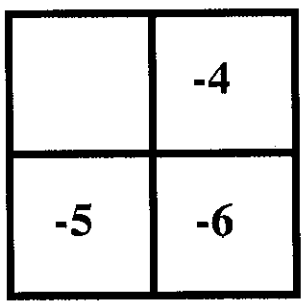

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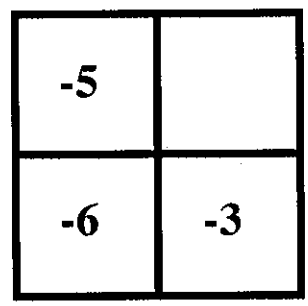

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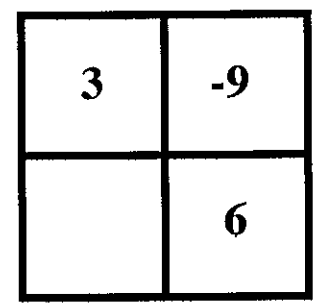

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Integer Two Ways # 1

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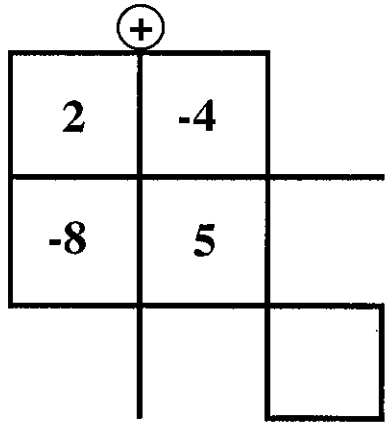

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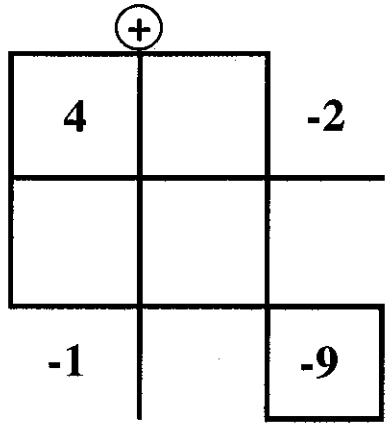

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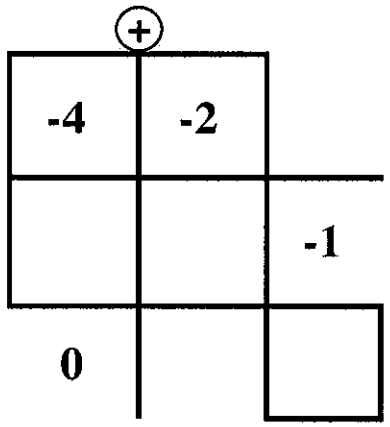

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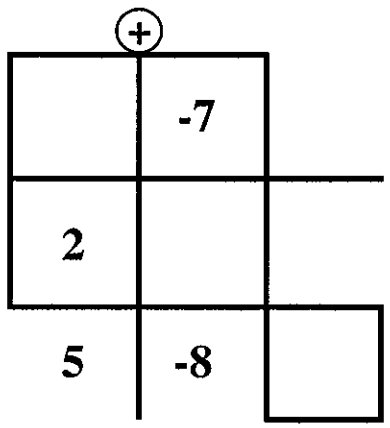

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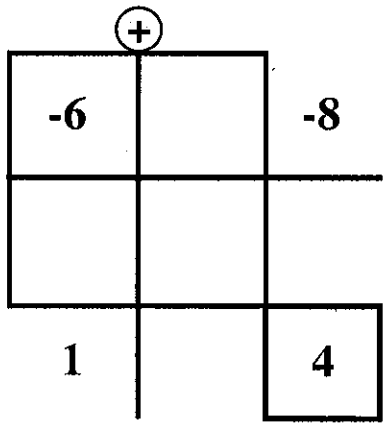

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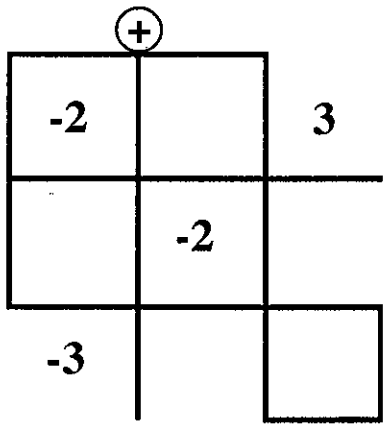

Fraction Balance # 1

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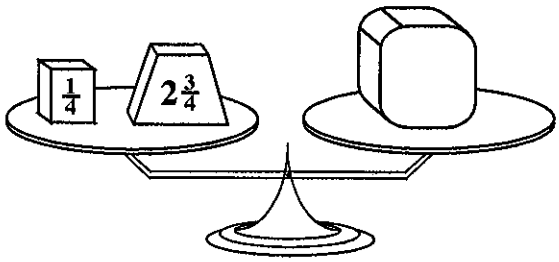

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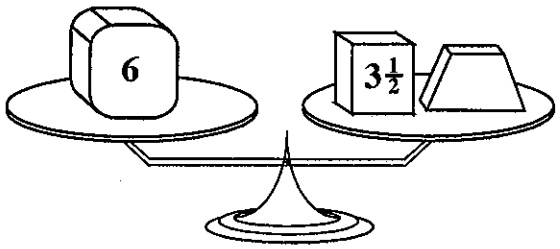
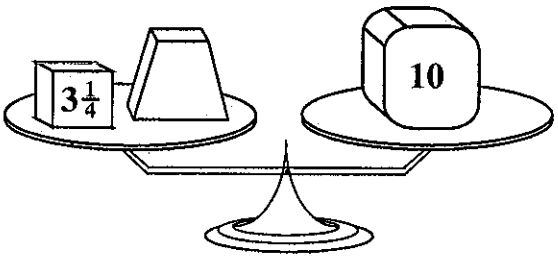


Boxes that are the same size and shape must have the same number in them.

3.



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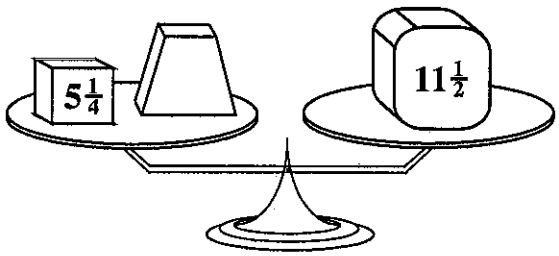

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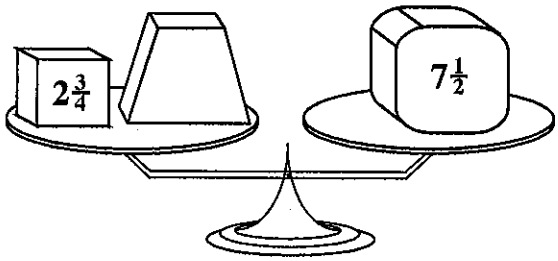

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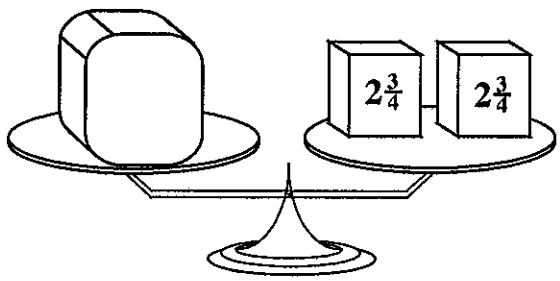

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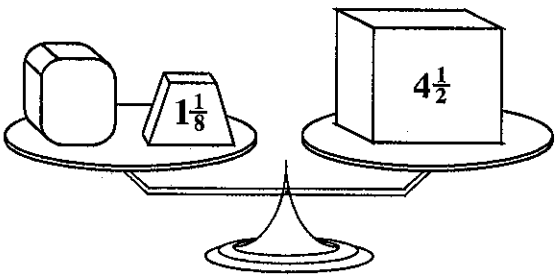

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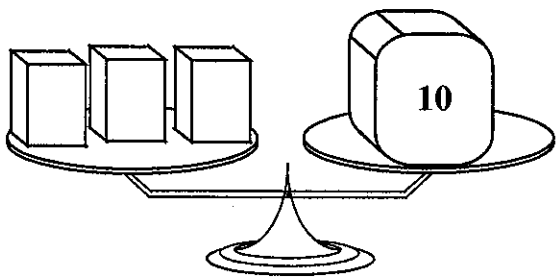

Fraction Math Squares # 1

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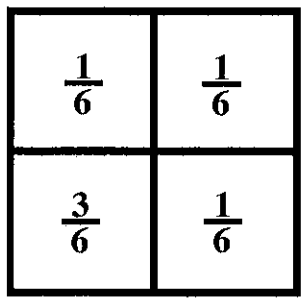

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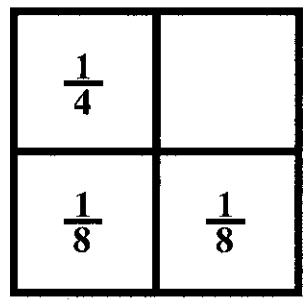
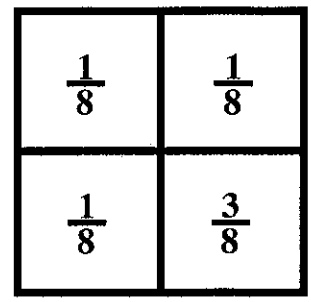


$\frac{3}{4}$

3.



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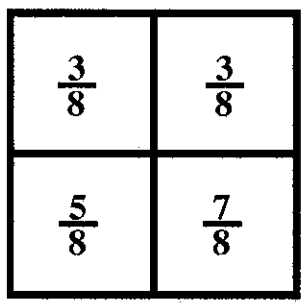

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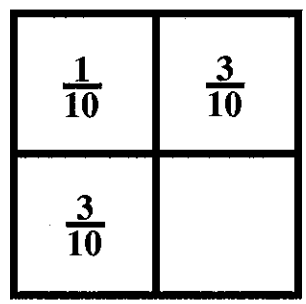

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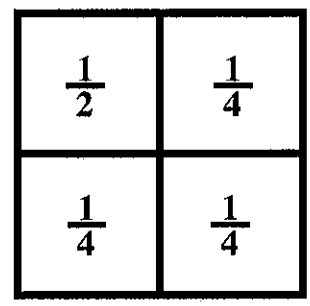

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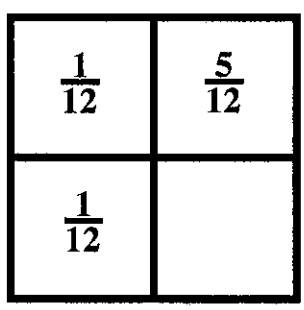
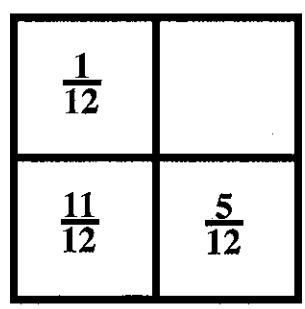


$1\frac{5}{12}$

8.



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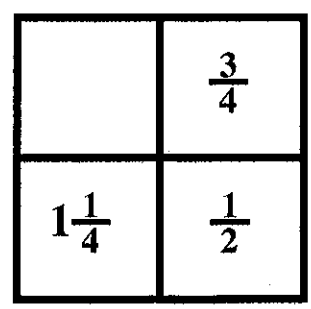

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Decimal Balances #2

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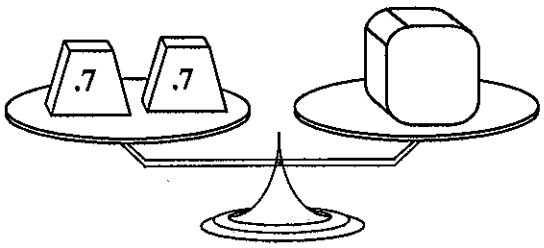

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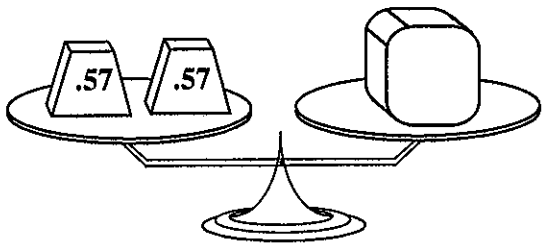
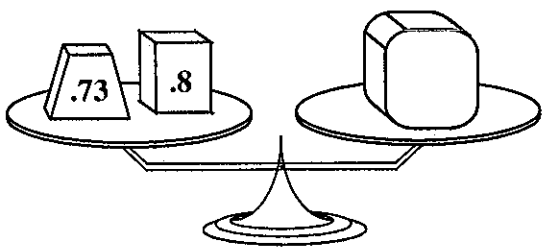


Boxes that are the same size and shape must have the same number in them.

3.



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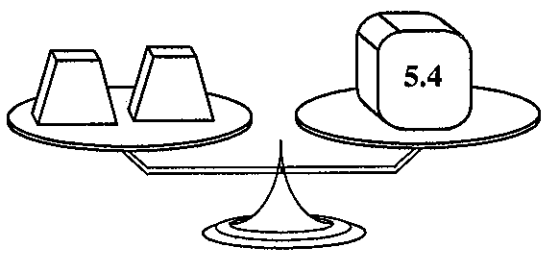

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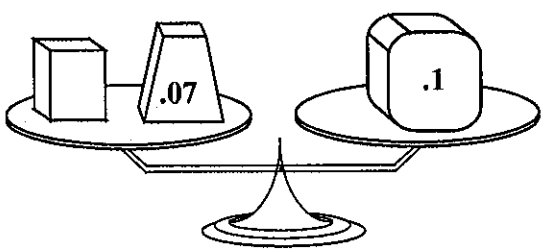

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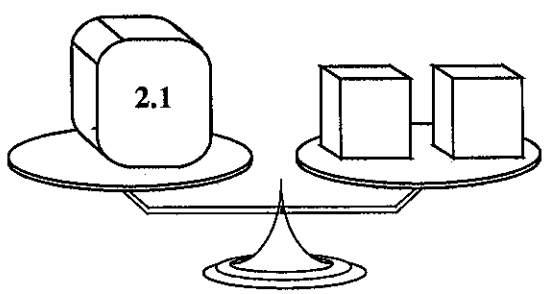

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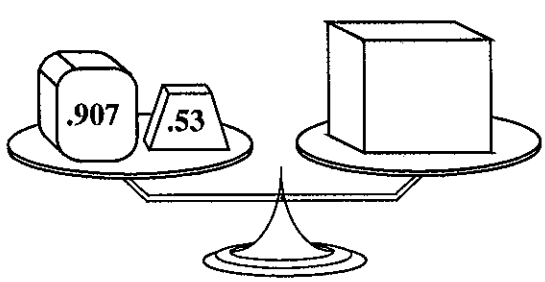

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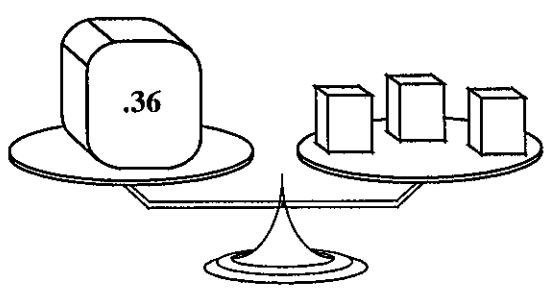

Decimal Two Ways # 1

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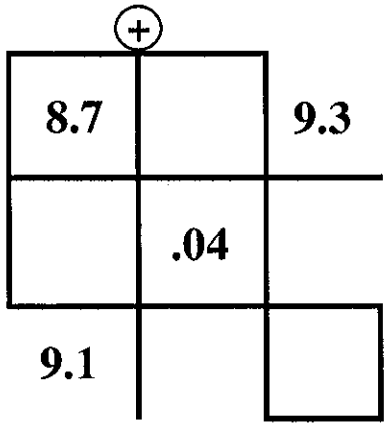

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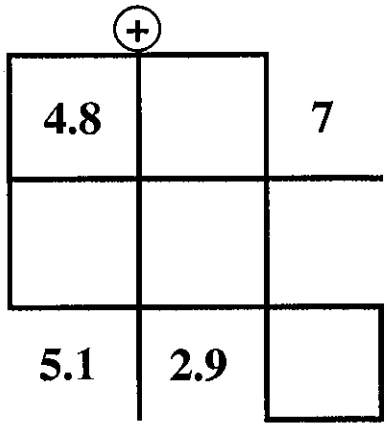

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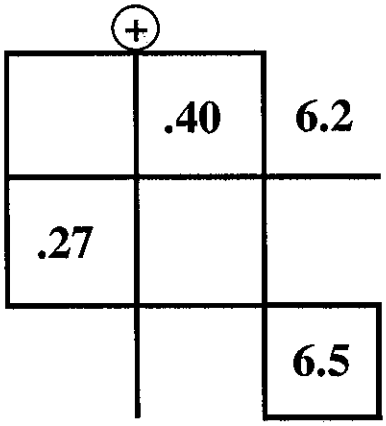

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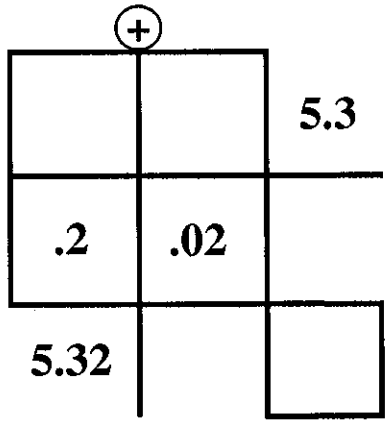

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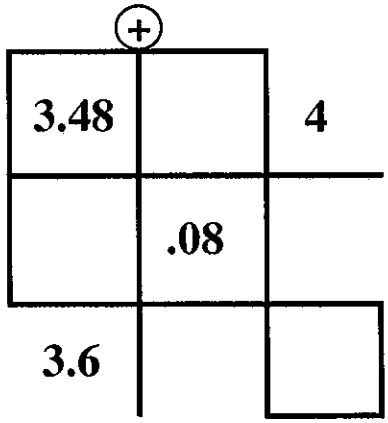

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