Below you will find outcomes from the elementary program of studies related to multiplicative thinking as the EMPL team sees it. Other outcomes may be closely related but were not included in this project.

+Grade Two

N 7	Illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]

+Grade Three

N 5	Illustrate, concretely and pictorially, the meaning of place value for numerals to 1000. [C, CN, R, V]
N 11	Demonstrate an understanding of multiplication to 5 × 5 by: • representing and explaining multiplication using equal grouping and arrays • creating and solving problems in context that involve multiplication • modeling multiplication using concrete and visual representations, and recording the process symbolically • relating multiplication to repeated addition • relating multiplication to division. [C, CN, PS, R] *Clarification:* Understand and recall multiplication facts to 5 × 5.
N 12	Demonstrate an understanding of division (limited to division related to multiplication facts up to 5 × 5) by: • representing and explaining division using equal sharing and equal grouping • creating and solving problems in context that involve equal sharing and equal grouping • modeling equal sharing and equal grouping using concrete and visual representations, and recording the process symbolically • relating division to repeated subtraction • relating division to multiplication. [C, CN, PS, R]

+Grade Four

N 1	Represent and describe whole numbers to 10 000, pictorially and symbolically. [C, CN, R, V]
N 4	Apply the properties of 0 and 1 for multiplication and the property of 1 for division. [C, CN, R]
N 5	Describe and apply mental mathematics strategies, such as: • skip counting from a known fact • using doubling or halving • using doubling or halving and adding or subtracting one more group • using patterns in the 9s facts • using repeated doubling to determine basic multiplication facts to 9 × 9 and related division facts. [C, CN, ME, R] *Clarification*: Understand and apply strategies for multiplication and related division facts to 9 × 9 Recall multiplication and related division facts to 7 × 7.
N 6	Demonstrate an understanding of multiplication (2- or 3-digit by 1-digit) to solve problems by: • using personal strategies for multiplication with and without concrete materials • using arrays to represent multiplication • connecting concrete representations to symbolic representations • estimating products • applying the distributive property. [C, CN, ME, PS, R, V]
N 7	Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by: • using personal strategies for dividing with and without concrete materials • estimating quotients • relating division to multiplication. [C, CN, ME, PS, R, V]

+Grade Five

N 1	Represent and describe whole numbers to 1 000 000. [C, CN, V, T]
N 3	Apply mental mathematics strategies and number properties, such as: • skip counting from a known fact • using doubling or halving • using patterns in the 9s facts • using repeated doubling or halving in order to understand and recall basic multiplication facts (multiplication tables) to 81 and related division facts. [C, CN, ME, R, V] *Clarification*: Understand, recall and apply multiplication and related division facts to 9 × 9.

N 4	Apply mental mathematics strategies for multiplication, such as: • annexing then adding zero • halving and doubling • using the distributive property. [C, CN, ME, R, V]
N 5	Demonstrate, with and without concrete materials, an understanding of multiplication (2-digit by 2-digit) to solve problems. [C, CN, PS, V]
N 6	Demonstrate, with and without concrete materials, an understanding of division (3-digit by 1-digit), and interpret remainders to solve problems. [C, CN, ME, PS, R, V]
P/R 3	Solve problems involving single-variable, one-step equations with whole number coefficients and whole number solutions. [C, CN, PS, R]

+Grade Six

N 1	Demonstrate an understanding of place value, including numbers that are: • greater than one million • less than one thousandth [C, CN, R, T]
N 2	Solve problems involving whole numbers and decimal numbers. [ME, PS, T]
N 3	Demonstrate an understanding of factors and multiples by: • determining multiples and factors of numbers less than 100 • identifying prime and composite numbers • solving problems using multiples and factors. [CN, PS, R, V]
N 4	Relate improper fractions to mixed numbers and mixed numbers to improper fractions. [CN, ME, R, V]
N 5	Demonstrate an understanding of ratio, concretely, pictorially and symbolically. [C, CN, PS, R, V]
N 6	Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically. [C, CN, PS, R, V]
N 8	Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors). [C, CN, ME, PS, R, V]

Evidence of Understanding

+Big Idea #1

Multiplicative thinking extends through place value, percentages, scale, proportions, rate, ratio, arrays, division, fractions, decimals, etc.

What to look for, what might be evidence of understanding?

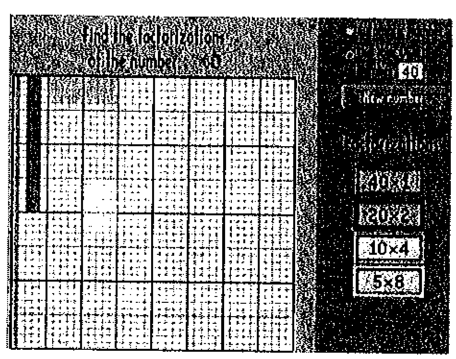
- Students understand place of a number, but not necessarily the value of the number. For example, in 324, the 2 is in the tens place. If you ask a student how many 10's are in 324, the correct answer is 32 tens.

+Big Idea #2

Multiplication can often be solved using repeated addition

What to look for, what might be evidence of understanding?

- Students see 3 x 4 as 3 groups of four and that this is different than 4 groups of three.
- Students can explain why 3 x 4 is equal to 4 x 3.
- Students can also provide situations showing 3 x 4 and 4 x 3 are different. For example, 3 dogs with four legs is not the same as 4 dogs with three legs.
- Students see 40 as:
  - 1 forty
  - 2 twenties
  - 4 tens
  - 5 eights
  - 8 fives
  - 10 fours
  - 20 two
  - 40 ones



Warning

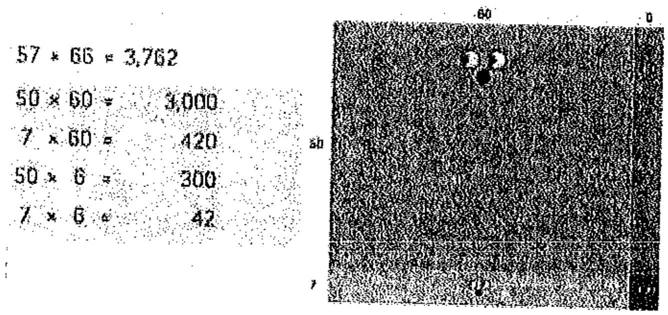
Multiplication cannot always be solved using repeated addition. For example, 2.5 x 3.8 and ½ x ¾ can not be interpreted as repeated addition. However, this concept comes later in grades 7 and up.

+Big Idea #3

The distributive property is a powerful strategy for mental math.

What to look for, what might be evidence of understanding?

- Students decompose numbers in an advantageous manner such as:
  - 28 x 30 = (20 x 30) + (8 x 30) or (30 x 30) - (2 x 30)
  - 12 x 45 = (10 + 2) x (40 + 5) = (10 x 40) + (10 x 5) + (2 x 40) + (2 x 5)
  - 57 x 66



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- Evidence from top-end students.
  - 57 x 66 = (60 - 3) x (70 - 4)

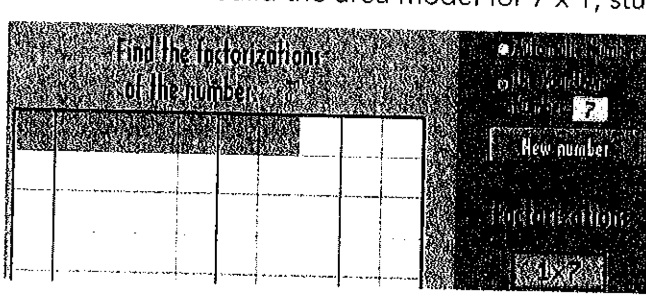
Watch a useful video on distributive property.*
**Please note that you can demonstrate the distributive property in the same manner as shown on the video using Lego blocks.*

+Big Idea #4

Multiplication and division by 1 and 0 have special properties.

What to look for, what might be evidence of understanding?

- When asked to build the area model for 7 x 1, student would build a line of 7.



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- When asked to build the area model for 7 x 0, student should say that this is not possible.

+Big Idea #5

Multiplication and division are inverse operations.

What to look for, what might be evidence of understanding?

- Given a pair of factors, students can
  - create the 4 multiplication and division statements that result. For example, 3 and 4: 3x 4 = 12; 4 x 3 = 12; 12 ÷ 4 = 3; 12 ÷ 3 = 4
- Students understand that commutative property only applies to multiplication but not to division. Example: 3 x 4 = 4 x 3 but 12 ÷ 4 ≠ 4 ÷ 12

Possible Resources

"Teacher Guidance for Multiplication and Division"
Contains many activities and models for teaching purposes.
https://www.learntogether.org.uk/Resources/Documents/Teaching%20Guidance%20Multi%20and%20Div[1].pdf

"Assessing Multiplicative Thinking Using Rich Tasks" – Dianne Siemon and Margarita BreedContains several tasks you could use to check student understanding.
http://www.aare.edu.au/data/publications/2006/sie06375.pdf

"Scaffolding Numeracy in the Middle Years" – Australian Research Council Linkage Project Contains Assessment materials, learning plans, and authentic tasks.
http://www.education.vic.gov.au/school/teachers/teachingresources/discipline/maths/assessment/pages/scaffoldnum.aspx

What do I need to know as a teacher in order to be able to teach the concept(s)?

+Potential Misunderstandings

The Question: (Focusing on the Misconception)	The Follow Up Question that will challenge the assumption.	Background Info
When you multiply 2 numbers, the product is bigger.	½ x 10 = ?	Misunderstanding: Students generalize this idea when they only see examples that have whole numbers for the multiplicand and multiplier, they only ever see answers that are bigger. Although only students in Grade 5 and higher are exposed to multiplication of decimals and fractions, students in lower grades can be exposed to simpler experiences with the idea. For example, "I have 10 cookies. I'm going to give ½ of them to my friend and keep ½ of them to myself."
When you divide 2 numbers, the quotient is smaller.	20 ÷ 0.5 = ?	Misunderstanding: Students generalize this idea when they only see examples that have whole numbers for the dividend and divisor, they only ever see answers that are smaller. Although only students in Grade 5 and higher are exposed to division of decimals and fractions, students in lower grades can be exposed to simpler experiences with the idea. For example, "I have 3 cookies. I'm going to split each in half so I can give half a cookie to each person. How many people can I feed?"

Division stops when you end up with a remainder. Example 17÷ 4 = 4R1	Case 1: If I need 17 kg of flour and I can buy 4 kg bags, how many bags do I need? Case 2: I have 17 chocolate bars and am sharing between 4 people. How much do we each get?	Case 1: Students need to understand that in certain situations, a remainder must interpreted appropriately. In this case, students must understand that they actually need 5 bags not 4 bags. Rounding up becomes necessary due to the context of the problem. Case 2: Students need to understand that in certain situations, a remainder can be divided some more. In this case, students must understand that a piece of the chocolate bar can be given to each person. Therefore, they are actually received 4 ¼ chocolate bars.
The associative property is possible with all 4 operations.	125 + (25 ÷ 5) equal to (125 + 25) ÷ 5 ? Is 6 ÷ (4 ÷ 2) equal to (6 ÷ 4) ÷ 2?	Misunderstanding: Students learn thataddition is associative so it doesn't matter if you do 2 + (4 + 3) or (2 + 4) + 3 because you get the same answer. They may transfer this concept to questions with mixed operations.
The commutative property is possible with all 4 operations.	Is 72 ÷ 9 equal to 9 ÷ 72 ?	Misunderstanding: Students learn that multiplication is commutative so it doesn't matter if you do 2 x 4 or 4 x 2 because you get the same answer. They may transfer this concept to division.
All units work on base 10.	What is 3 ½ hours in minutes?	Time works on a different base. Therefore the answer is not 350 minutes.
For teacher knowledge only: Multiplicative Thinking is repeated addition.	0.5 x 1.6	Misunderstanding: "While repeated addition may be an appropriate beginning, to maintain that interpretation of multiplication is ultimately disabling because it does not provide children with important multiplicative structures. Multiplicative thinking cannot be generalised in any simple way from additive thinking. Unless teachers consciously help children develop multiplicative thinking, which goes well beyond repeated addition, it may not happen for many children." Source

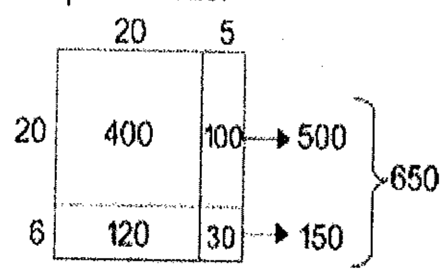
Central Alberta Regional Consortium · Southern Alberta Professional Development Consortium · Learning Network Educational Services · Consortium provincial francophone · Edmonton Regional Learning Consortium · Calgary Regional Consortium · NRLC Regional Learning

What do I need to know as a teacher in order to be able to teach the concept(s)?

+Vocabulary

Area Model / Rectangular Array

- example: 26 X 25:



Associative Property

- you can multiply regardless of how the numbers are grouped (using parenthesis)
  - 3 x 4 x 2 = (3 x 4) x 2 or 3 x (4 x 2)
  - Does not apply to division! 6 ÷ (4 ÷ 2) is not equal to (6 ÷ 4) ÷ 2

Cartesian Product aka "for each"

- For each ten, there are ten ones; for each hundred there are 10 tens; for each one there are 10 tenths
- I have 3 shirts and 2 shorts. How many outfits do I have? Solution: For each shirt, there are 2 shorts. So there are 6 outfits altogether.

Commutative Property

- two numbers can be multiplied in either order. Example: 3 x 4 or 4 x 3

Composite Number

- A composite number is a positive integer that has at least one positive divisor other than one or the number itself. In other words, a composite number is any integer greater than one that is not a prime number. For example, 9 is divisible by 3 so it is composite. The number 1 is neither prime nor composite.

Distributive Property / Partitioning

- A number in a multiplication expression can be decomposed into two or more numbers. The distributive property can involve:
  - multiplication over addition (e.g., 6 x 47 = (6 x 40) + (6 x 7))
  - multiplication over subtraction (e.g. 4 x 98 = (4 x 100) - (4 x 2))
  - division over addition (e.g. 72 / 6 = (60 / 6) + (12 / 6))
  - division over subtraction (4700 / 4 = (4800 / 4) - (100 / 4))
  - 24 is 2 twelves, 3 eights, 4 sixes, 6 fours, and 12 twos
  - Distributive Law: "the multiplication operation may be applied to a number which has been partitioned without altering the outcome."
  - 3 x 6 = 3 x (2 + 4) = (3 x 2) + (3 x 4)
  - 3 x (4 - 2) = (3 x 4) - (3 x 2)
  - "Distributive Property" Video

Factor

- a factor is a term that exactly divides a given term. Example, 2 is a factor of 2 because you can divide 12 by 2 and end up with an answer that is not a fraction.

Factor-factor-product

- 24 is 2 x 2 x 2 x 3

Greatest Common Divisor/Factor

- The greatest common factor (gcf) is the largest natural number that exactly divides two or more given natural numbers.

Inverse Operation

- "The operation which is 'opposite' mathematically to that being considered. Thus, division is the inverse of multiplication and vice versa."

Iteration

- the act of repeating a process in order to reach a desired goal. For example: using a metre stick in order to measure the length of the classroom; measuring the height of a horse in hands;

Least/Lowest Common Multiple

- The least/lowest common multiple (lcm) is the smallest natural number that is a multiple of two or more given natural numbers.

Multiple

- The result of multiplying a number by an integer (not by a fraction). For example, if you start with the number 3 and multiply it by 4 to get 12, you say that 12 is a multiple of 3. It is also a multiple of 4 because you multiplied 4 by 3 to get 12.

Multiplicative Comparison

- Two quantities compared on the basis of 'as many as'. For example. Paul has 4 apples. Mary has 3 times as many as Paul.

Operation Terms

- 3 x 2 = 6
  - 3 = multiplicand
  - 2 = multiplier
  - 6 = product
- 6 ÷ 2 = 3
  - 6 = dividend
  - 2 = divisor
  - 3 = quotient

Rate

- A rate is a two-term ratio used to compare quantities having different units. Example: 75 km/hr, $10 for 2 books.

Ratio

- A ratio is a comparison of numbers or quantities. Example: 2:1 may mean for every 2 slices of pizza my dad eats, I eat 1.

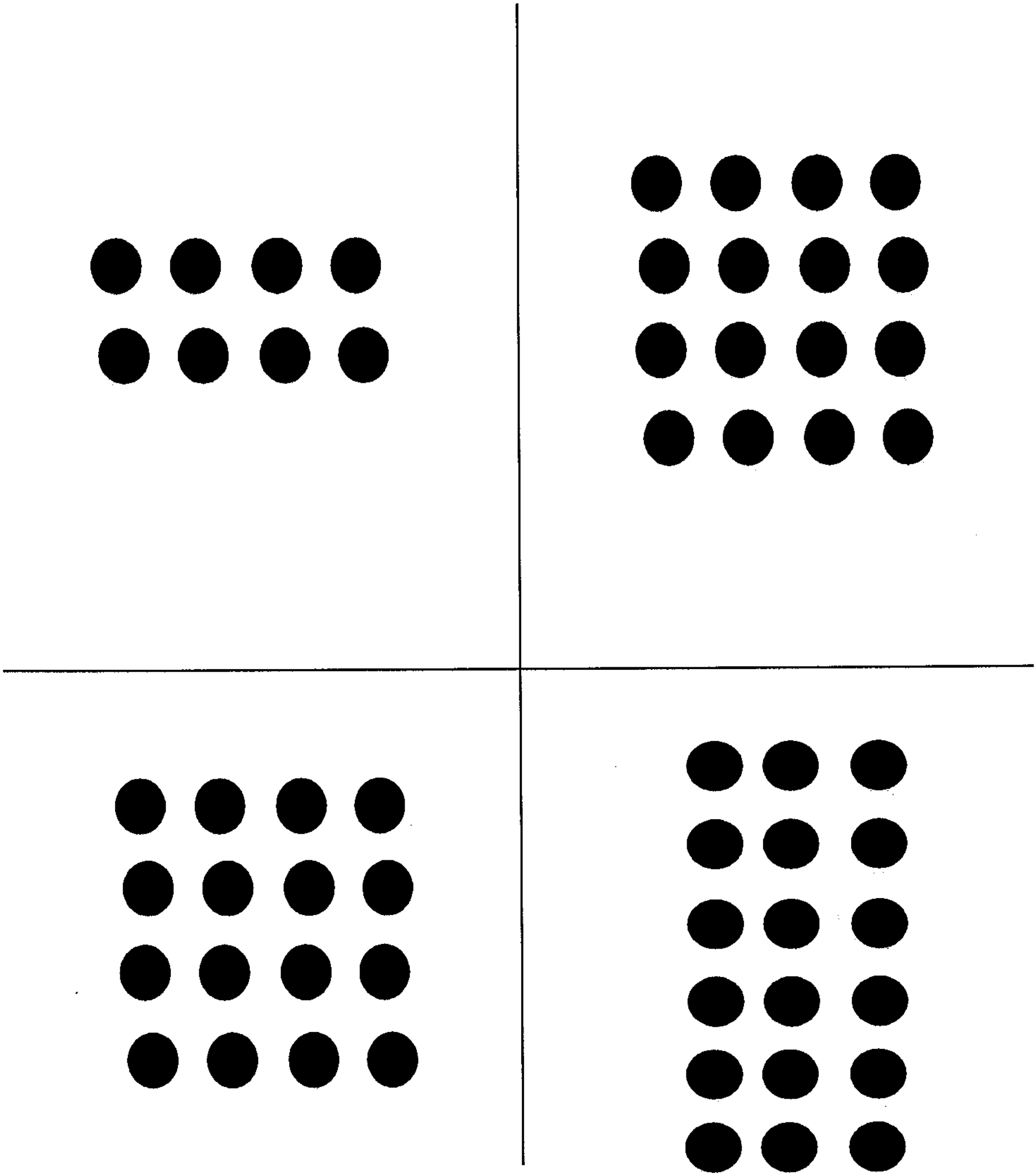
Prime Number

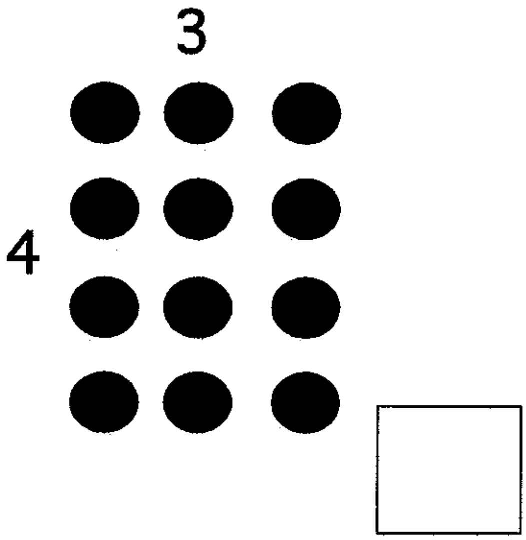
- A prime number is a natural number that has exactly two factors: one and itself. For example, 11 is prime because it can not be divided by any number other than 11 and 1. The number 1 is neither prime nor composite.

Further Resources:

Math is Fun - Illustrated Mathematics Dictionary – Mathematics Vocabulary & Illustrations
https://www.mathsisfun.com/definitions/index.html

Mathematics Glossary - LearnAlberta.ca
Example Link - Associative Property
http://www.learnalberta.ca/content/memg/division04/associative%20property/index.html

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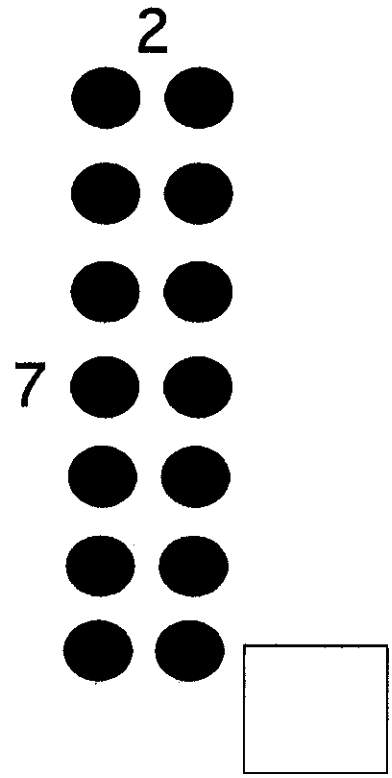
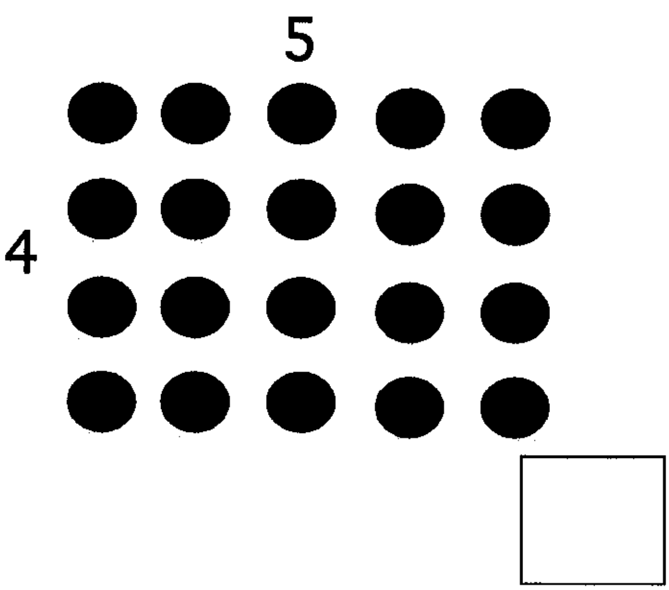
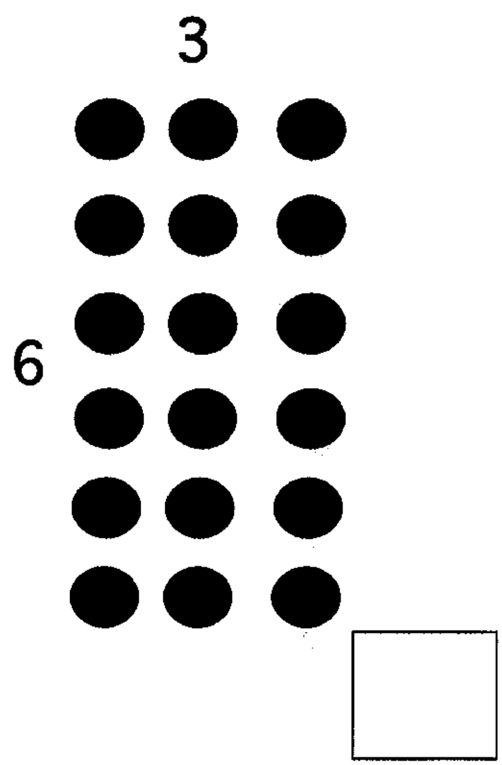
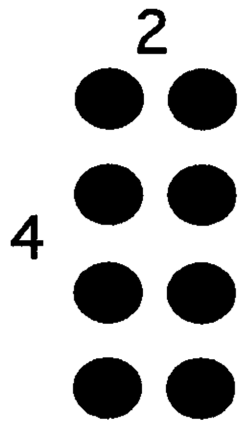
4

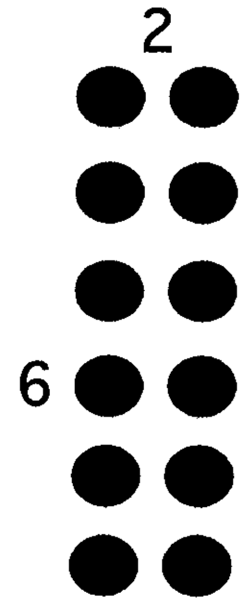

2
7


5
4

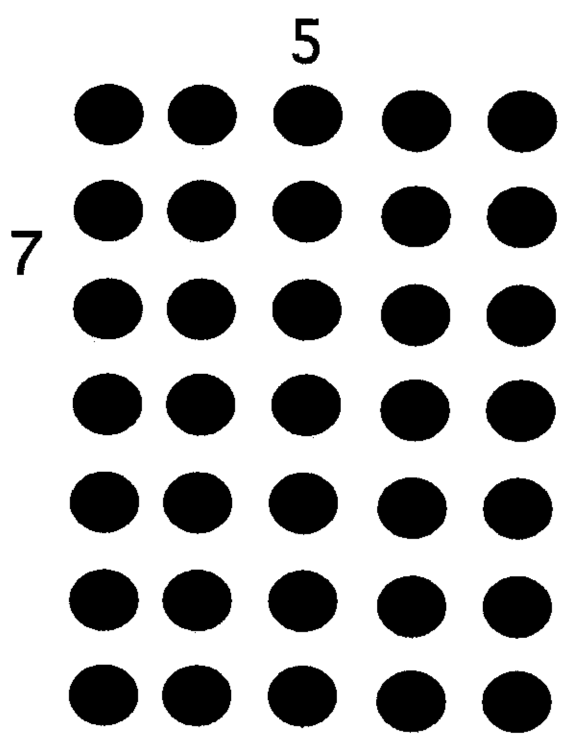

3
6



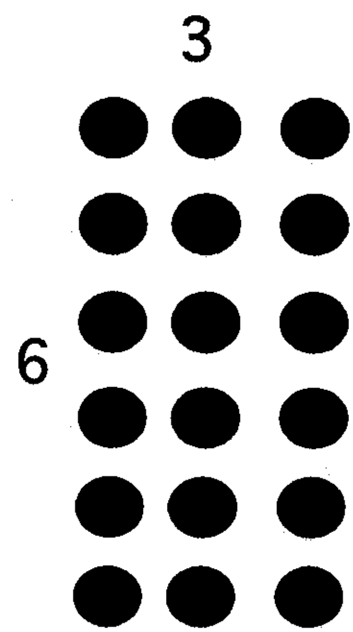
4 x 2 = ____



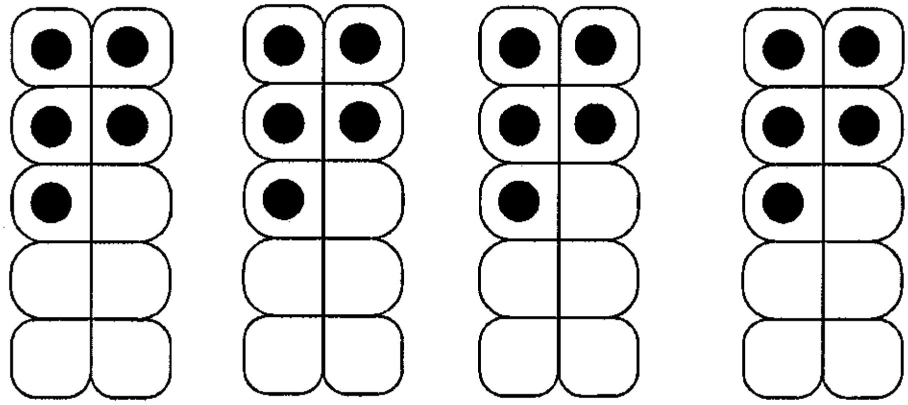
6 x 2 = _____

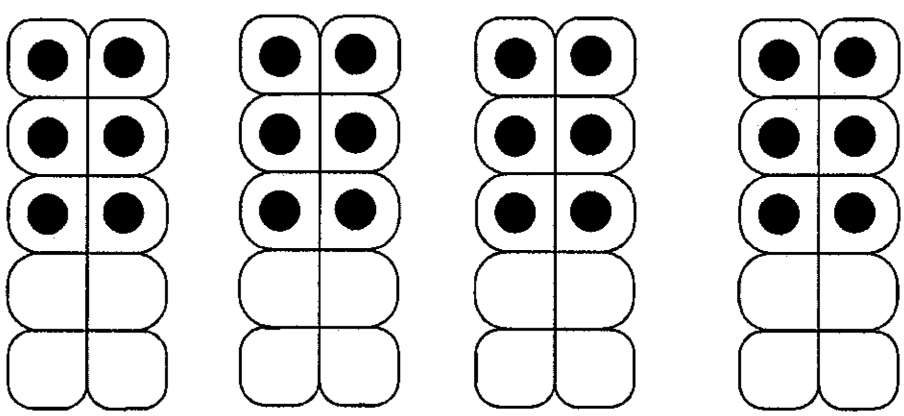


7 x 5 = ____

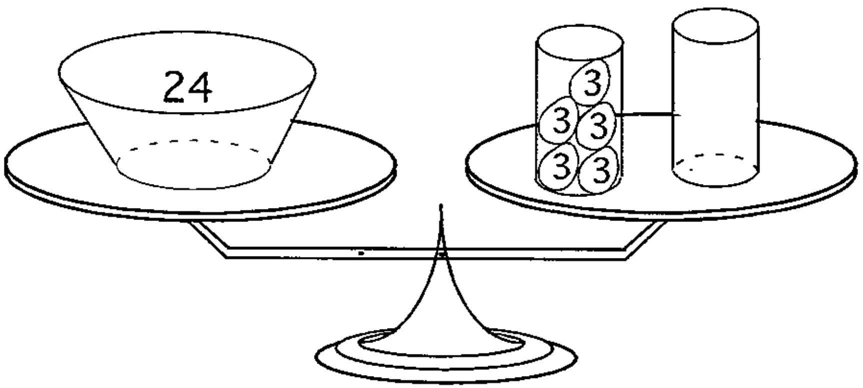


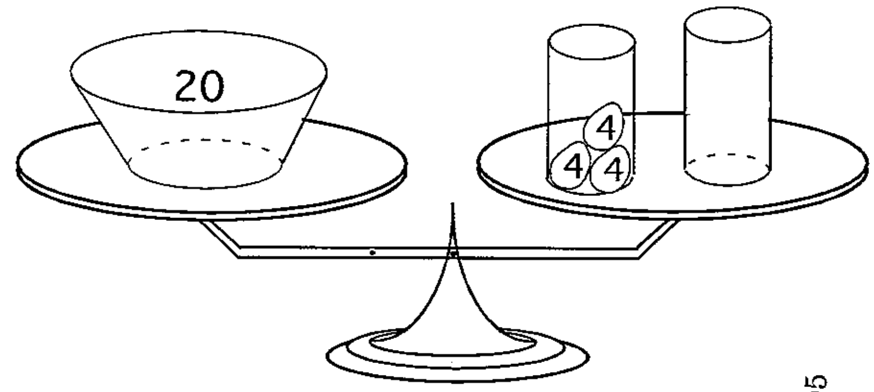
6 x 3 =____

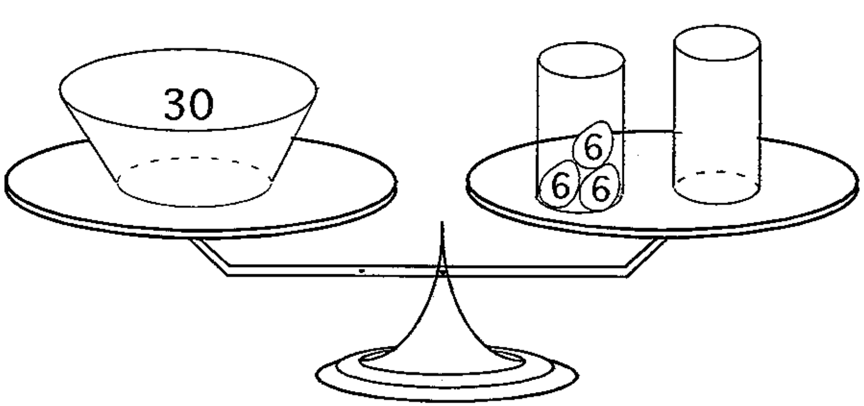


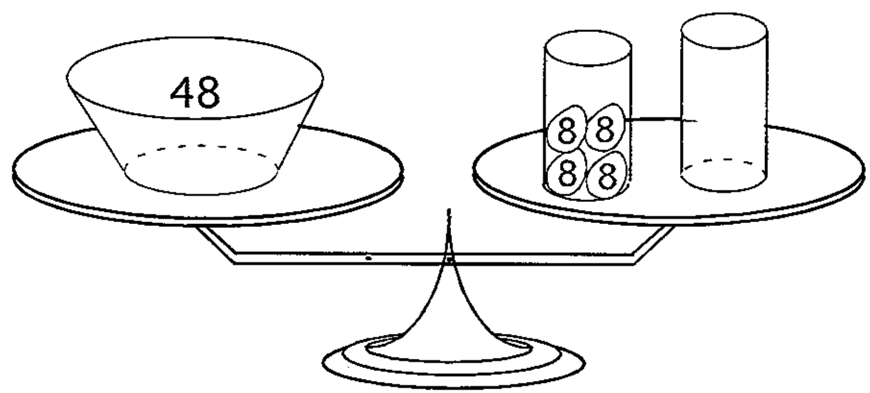


Draw in the number of bags needed to make the scale balance.



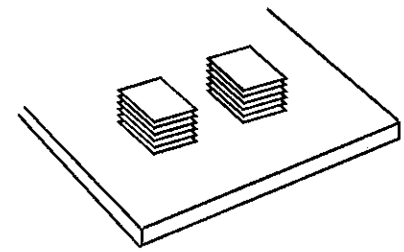




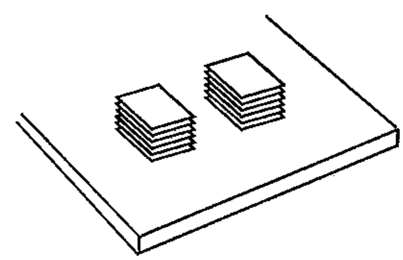


Stacking Cards

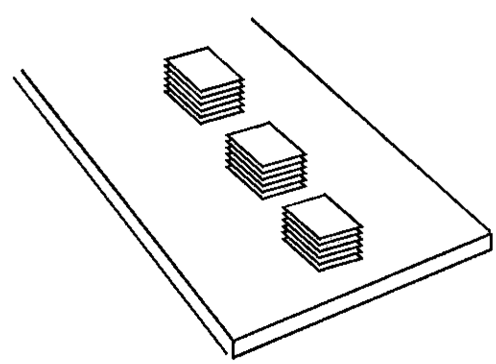
1. Chan was making stacks of baseball cards. He wanted 9 in each stack. How many stacks would there be with 36 cards? 45 cards?



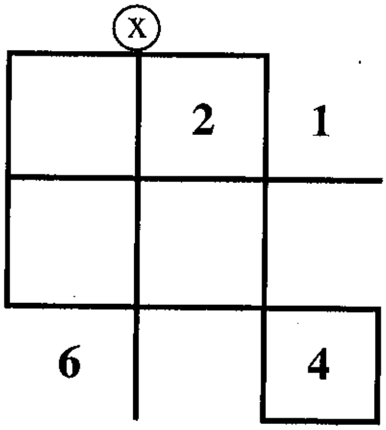
2. Suppose he wanted to make 12 stacks with 60 cards. How many cards would be in each stack?



3. After making six stacks of nine cards each, Chan added more cards to make seven stacks with ten cards in each stack. How many MORE cards did he use?



Fraction Two Ways # 1

1.


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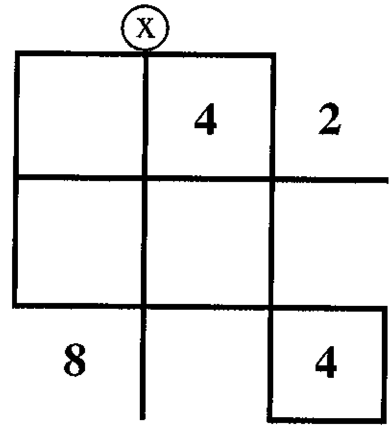

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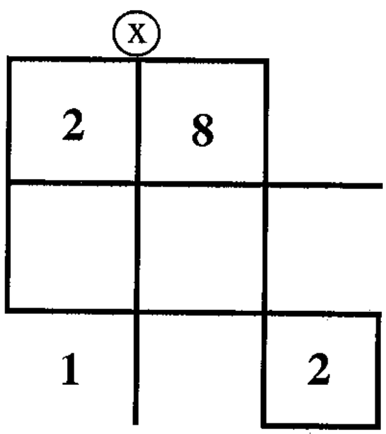

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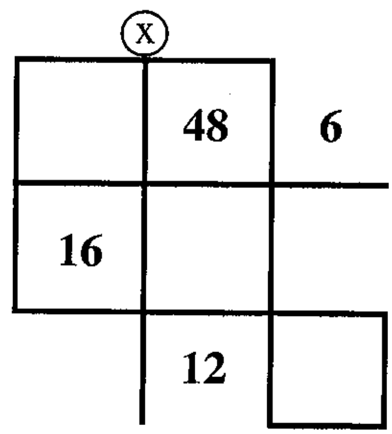

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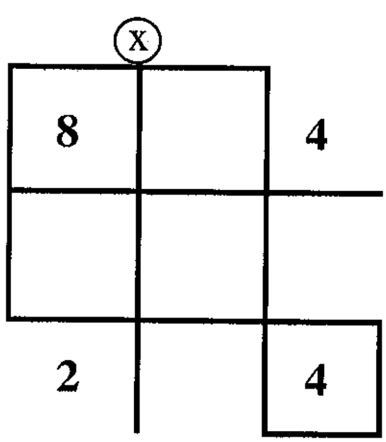

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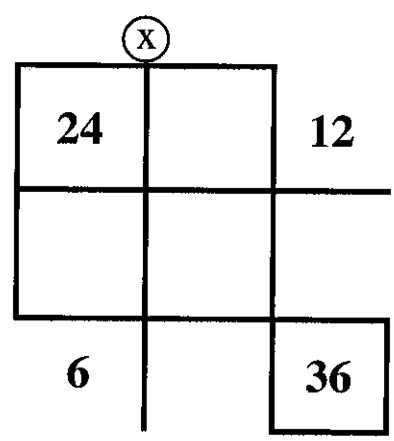


Integer Two Ways # 5

1. X

	-1	1
-1		-1

2. X

-2	-1	
3	5	

3. X

	-2	-4
-2		-8

4. X

-3		-6
	-4	-12

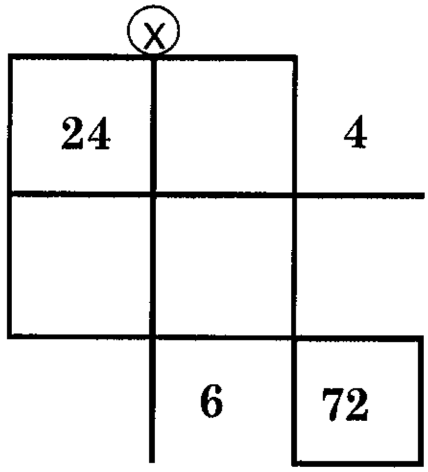
5. X

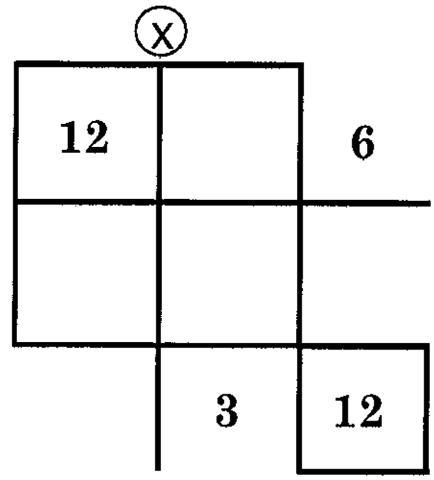
-6		
	2	-2
		-36

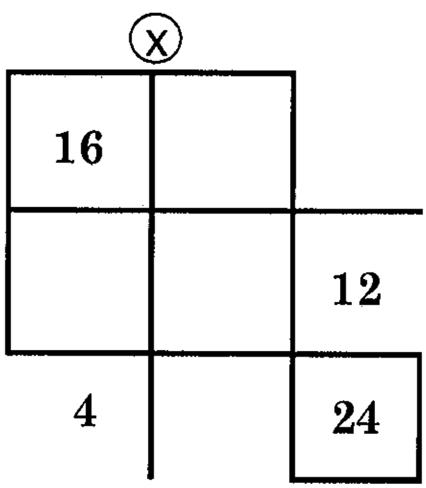
6. X

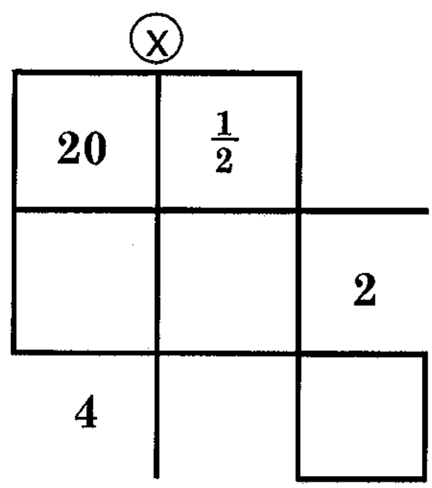
-1		2
5		-20

Two Ways









Decimals Two Ways # 5

1. (X)

100	.35	
.05	4	

2. (X)

50	.02	
.04		.12

3. (X)

	.2	
.25		2
1		

4. (X)

4	.15	
	.3	3

5. (X)

15	.2	
.3		
		9

6. (X)

40	.08	
.25	20	

Percent Benchmarks # 1

1.

100%	50%	25%	10%	5%	$2\frac{1}{2}$%	1%
1000						

a) 75% x 1000 = ______ b) 15% of 1000 is ______

c) 35% x 1000 = ______ d) 60% of 1000 is ______

e) 20% x 1000 = ______ f) 30% of 1000 is ______

g) 51% x 1000 = ______ h) 26% of 1000 is ______

i) 99% x 1000 = ______ j) 105% of 1000 is ______

Make up two of your own.

k) ______ x 1000 = ______ l) ______ of 1000 is ______

2.

100%	50%	25%	10%	5%	$2\frac{1}{2}$%	1%
$80						

a) 30% of $80 = ______ b) 60% x $80 is ______

c) 90% of $80 = ______ d) 15% x $80 is ______

e) 35% of $80 = ______ f) 70% x $80 is ______

g) 40% of $80 = ______ h) 75% x $80 is ______

i) 125% of $80 = ______ j) 24% x $80 is ______

Make up two of your own.

k) ______ of $80 = ______ l) ______ x $80 is ______